Answer on Question #72711 – Math – Statistics and Probability

Question

Changes in airport procedures require considerable planning. Arrival rates of aircraft are important factors that must be taken into account. Suppose small aircraft arrive at a certain airport, according to a Poisson process, at the rate of 6 per hour. Thus, the Poisson parameter for arrivals over a period of hours is $\mu = 6t$.
(a) What is the probability that exactly 4 small aircraft arrive during a 1-hour period?
(b) What is the probability that at least 4 arrive during a 1-hour period?
(c) If we define a working day as 12 hours, what is the probability that at least 75 small aircraft arrive during a working day?

Solution

a) The probability that exactly 4 small aircrafts arrive during a 1-hour period is calculated using Poisson distribution with $\mu = 6$ airplanes/hour

$$P(X = 4) = \frac{e^{-6}(6)^4}{4!} = 0.13385$$

b) The probability that at least 4 small aircrafts arrive during a 1-hour period is

$$P(X \geq 4) = 1 - P(X \leq 3) =$$
$$= 1 - (P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3))$$
$$= 1 - \left(\frac{e^{-6}(6)^0}{0!} + \frac{e^{-6}(6)^1}{1!} + \frac{e^{-6}(6)^2}{2!} + \frac{e^{-6}(6)^3}{3!}\right) =$$
$$= 1 - (0.00248 + 0.01487 + 0.04462 + 0.08924) = 0.84879$$

c) The probability that at least 75 small aircrafts arrive during a day is calculated using a Poisson distribution with $\mu = 6(12) = 72$ airplanes/day

$$P(X \geq 75) = 1 - P(X \leq 74) = 1 - \sum_{x=0}^{74} \frac{e^{-6}(72)^x}{x!} = 1 - 0.62267 = 0.37733$$

Answer: a) 0.13385; **b)** 0.84879; **c)** 0.37733.