

Answer on Question #72710 - Math - Statistics and Probability

Question:

An automobile manufacturer is concerned about a fault in the braking mechanism of a particular model. The fault can, on rare occasions, cause a catastrophe at high speed. The distribution of the number of cars per year that will experience the catastrophe is a Poisson random variable with $\lambda = 5$.

- (a) What is the probability that at most 3 cars per year will experience a catastrophe?
- (b) What is the probability that more than 1 car per year will experience a catastrophe?

Solution:

Let ξ be a random variable representing the number of cars per year that will experience the catastrophe, ξ is a Poisson random variable with $\lambda = 5$. It means

$$\Pr(\xi = k) = \frac{5^k}{k!} e^{-5}, k \in \{0, 1, 2, \dots\}.$$

- (a) The probability that at most 3 cars per year will experience a catastrophe is equal to

$$\Pr(\xi \leq 3) = \sum_{k=0}^3 \frac{5^k}{k!} e^{-5} = e^{-5} \left(1 + 5 + \frac{25}{2} + \frac{125}{6} \right) \approx 0.265.$$

- (b) The probability that more than 1 car per year will experience a catastrophe is equal to

$$\Pr(\xi > 1) = 1 - \Pr(\xi \leq 1) = 1 - \sum_{k=0}^1 \frac{5^k}{k!} e^{-5} = 1 - e^{-5}(1 + 5) \approx 0.960.$$

Answer: (a) ≈ 0.265 ; (b) ≈ 0.960 .