Question

Find the mean and variance of the random variable X in Exercise 5.61, representing the number of persons among 10,000 who make an error in preparing their income tax returns.

Solution

Suppose that, on average, 1 person in 1000 makes a numerical error in preparing his or her income tax return. If 10,000 returns are selected at random and examined, find the mean and variance of the random variable X.

We have n = 10000 is large and p = 1/1000 is near 0, then the binomial distribution can be approximated by the Poisson distribution with parameter $\lambda = np = 10000 \times (1/1000) = 10$.

Use the Poisson distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, ...$$

The mean and the variance of the Poisson distribution are both equal to λ

 $\mu = E(X) = \lambda,$ $V(X) = \sigma^{2} = \lambda,$ $\mu = E(X) = 10,$ $V(X) = \sigma^{2} = 10.$ Answer: $\mu = E(X) = 10, V(X) = \sigma^{2} = 10$

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