Answer on Question #72707, Math / Statistics and Probability

The probability that a student at a local high school fails the screening test for scoliosis (curvature of the spine) is known to be 0.004. Of the next 1875 students at the school who are screened for scoliosis, find the probability that

(a) fewer than 5 fail the test;

(b) 8, 9, or 10 fail the test.

Solution

Let X be the random variable representing the number of students who fail out of the next 1875 students.

If a student fails the screening test, we consider that as a success.

Then $p = 0.004$.

Trials are independent.

Hence, X has a binomial distribution with parameters $n = 1875$ and $p = 0.004$.

$$X \sim Bin(n, p), \quad \text{where } n = 1875 \text{ and } p = 0.004$$

We have $n = 1875$ is large and $p = 0.004$ is near 0, then the binomial distribution can be approximated by the Poisson distribution with parameter

$$\mu = np = 1875 \times 0.004 = 7.5 < 10$$

The Poisson distribution is a limiting case of the binomial distribution which arises when the number of trials n increases indefinitely whilst the product $\mu = np$, which is the expected value of the number of successes from the trials, remains constant.

Use Poisson distribution with $p.m.f\ p(x;\ \mu) = \frac{e^{-\mu}\mu^x}{x!} \qquad x = 0, 1, 2, \dots$

$$P(X = x) = \frac{e^{-\mu}\mu^x}{x!} \qquad x = 0, 1, 2, \dots$$

(a) Fewer than 5 fail the test

$P(X < 5) = P(X \le 4) =$

$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) =$

$= \frac{e^{-7.5}7.5^0}{0!} + \frac{e^{-7.5}7.5^1}{1!} + \frac{e^{-7.5}7.5^2}{2!} + \frac{e^{-7.5}7.5^3}{3!} + \frac{e^{-7.5}7.5^4}{4!} =$

$= 0.000553 + 0.004148 + 0.015555 + 0.038889 + 0.072916 =$

$= 0.132061$

(b) 8, 9, or 10 fail the test

$P(X = 8) + P(X = 9) + P(X = 10) =$

$= \frac{e^{-7.5}7.5^8}{8!} + \frac{e^{-7.5}7.5^9}{9!} + \frac{e^{-7.5}7.5^{10}}{10!} =$

$= 0.13733 + 0.11444 + 0.08583 = 0.33760$