Answer on Question #72704, Math / Statistics and Probability.

Task. Suppose the probability that any given person will believe a tale about the transgressions of a famous actress is 0.8. What is the probability that

(a) the sixth person to hear this tale is the fourth one to believe it?

(b) the third person to hear this tale is the first one to believe it?

Solution.

(a) This is Negative Binomial Distribution.

If repeated independent trials can result in a success with probability p and a failure with probability $q=1-p$, then the probability distribution of the random variable X, the number of the trial on which the k th success occurs is

$$P(n;k,p)=\binom{n-1}{k-1}p^k q^{n-k}.$$

So, $n=6$, $k=4$, $p=0.8$ and $q=1-0.8=0.2$. Then the probability that the sixth person to hear this tale is the fourth one to believe it is

$$P(6;4,0.8)=\binom{6-1}{4-1}\cdot 0.8^4\cdot 0.2^{6-4}=\binom{5}{3}\cdot 0.8^4\cdot 0.2^2=\binom{5}{3}\cdot 0.8^4\cdot 0.2^2=\frac{5!}{3!\cdot 2!}\cdot 0.4096\cdot 0.04=$$

$$=10\cdot 0.4096\cdot 0.04=0.16384.$$

(b) In this case we have $n=3$, $k=3$, $p=0.8$ and $q=0.2$. Then the probability that the third person to hear this tale is the first one to believe it is

$$P(3;1,0.8)=\binom{3-1}{1-1}\cdot 0.8^1\cdot 0.2^{3-1}=\binom{5}{3}\cdot 0.8^4\cdot 0.2^2=\binom{2}{0}\cdot 0.8\cdot 0.04=1\cdot 0.8\cdot 0.04=0.032.$$

It is possible in another way:

$$P(No\cdot No\cdot Yes)=0.2\cdot 0.2\cdot 0.8=0.032.$$

Answer: (a) 0.16384; (b) 0.032.