Answer on Question #72701, Math / Statistics and Probability
The probability that a student pilot passes the written test for a private pilot's
license is 0.7. Find the probability that a given student will pass the test
(a) on the third try;
(b) before the fourth try.
Solution
Let us consider that a written test for a pilot's license is a trial. Trials are
independent. If a student passes the test, we consider that as a success.
Since the person needs to pass the test only once, we are looking for the number of
trials to obtain one success, i.e. the Geometric distribution.
Let $X$ be the random variable representing the number of tries to pass the test, that
is $X$ is the number of trials required to get $1^{st}$ success. This is a negative binomial
or geometric variable.
$p = P(success) = 0.7$ and the number of successes to observe $r = 1$.
$$P(X = x) = b^*(x; r, p) \text{ for } x = r, r + 1, r + 2, \dots$$
$$P(X = x) = \binom{x-1}{r-1} p^r q^{x-r}$$
The geometric distribution is a special case of the negative binomial distribution
such that $r = 1$.
 (a) on the third try
The probability $P$ that the student passes on the third try
$$P(X = 3) = \binom{3-1}{1-1}(0.7)^1(1-0.7)^{3-1} = 0.063$$
(b) before the fourth try
$$P(X \leq 3) = p + p(1-p) + p(1-p)^2$$
$$P(X \leq 3) = 0.7 + 0.7(1-0.7) + 0.7(1-0.7)^2 = 0.973$$