Question #72662, Math / Calculus | for confirmation

Find the antiderivative of the following:

1. ((3x^2+14x+13)/(x+4) dx
2. ((2x+5)/(3x-1) dx
3. ((x^5-2x^3-2x)/(x^2+1) dx
4. ((x^3+4x)/(x^2-1) dx

Solution

1.

$$\int \frac{3x^2 + 14x + 13}{x + 4} dx$$

Substitute $u = x + 4 \ \rightarrow x = u - 4, \ dx = du$

$$\int \frac{3(u - 4)^2 + 14(u - 4) + 13}{u} du = \int \frac{3u^2 - 10u + 5}{u} du = \int (3u - 10 + \frac{5}{u}) du =$$

$$= 3 \int u du - 10 \int du + 5 \int \frac{du}{u} = 3\frac{u^2}{2} - 10u + 5\ln|u| + C =$$

Undo substitution $u = x + 4$

$$= \frac{3(x + 4)^2 - 20(x + 4) + 10\ln|x + 4|}{2} + C = \frac{3x^2 + 4x + 10\ln|x + 4| - 32}{2} + C$$

$$= \frac{3x^2}{2} + 5\ln|x + 4| + 2x + C$$

Answer:

$$\int \frac{3x^2 + 14x + 13}{x + 4} dx = \frac{3x^2}{2} + 5\ln|x + 4| + 2x + C$$

2.

$$\int \frac{2x + 5}{3x - 1} dx$$

Substitute $u = 3x - 1 \ \rightarrow x = \frac{u+1}{3}, \ dx = \frac{1}{3} du$

$$\int \frac{2(u + 1) + 15}{9u} du = \int \frac{2u + 17}{9u} du = \int (\frac{2}{9} + \frac{17}{9u}) du == \frac{2}{9} \int du + \frac{17}{9} \int \frac{du}{u}$$

$$= \frac{2}{9} u + \frac{17}{9} \ln|u| + C =$$

Undo substitution $u = 3x - 1$

$$= \frac{2(3x-1)+17\ln|3x-1|}{9} + C = \frac{6x+17\ln|3x-1|}{9} - 1 + C$$
$$= \frac{17}{9}\ln|3x-1| + \frac{2x}{3} + C$$

Answer:

$$\int \frac{2x+5}{3x-1}dx = \frac{17}{9}\ln|3x-1| + \frac{2x}{3} + C$$

3.

$$\int \frac{x^5-2x^3-2x}{x^2+1}dx = \int \frac{x(x^4-2x^2-2)}{x^2+1}dx$$

Substitute $u = x^2+1 \rightarrow x^2 = u-1,\ dx = \frac{1}{2x}du$

$$\int \frac{(u-1)^2-2(u-1)-2}{2u}du = \int \frac{u^2-4u+1}{2u}du = \frac{1}{2}\int (u-4+\frac{1}{u})du =$$
$$= \frac{1}{2}\int udu - 2\int du + \frac{1}{2}\int \frac{du}{u} = \frac{u^2}{4} - 2u + \frac{1}{2}\ln|u| + C =$$

Undo substitution $u = x^2+1$

$$= \frac{(x^2+1)^2-8(x^2+1)+2\ln|x^2+1|}{4} + C = \frac{x^4-6x^2+2\ln|x^2+1|-7}{4} + C$$
$$= \frac{x^4-6x^2+2\ln|x^2+1|}{4} + C$$

Answer:

$$\int \frac{x^5-2x^3-2x}{x^2+1}dx = \frac{x^4-6x^2+2\ln|x^2+1|}{4} + C$$

4.

$$\int \frac{x^3+4x}{x^2-1}dx = \int \frac{x(x^2+4)}{x^2-1}dx$$

Substitute $u = x^2-1 \rightarrow x^2 = u+1,\ dx = \frac{1}{2x}du$

$$\int \frac{(u+1)+4}{2u}du = \int \frac{u+5}{2u}du = \frac{1}{2}\int (1+\frac{5}{u})du == \frac{1}{2}\int du + \frac{5}{2}\int \frac{du}{u}$$
$$= \frac{u}{2} + \frac{5}{2}\ln|u| + C =$$

Undo substitution $u = x^2-1$

$$= \frac{(x^2-1)+10\ln|x^2-1|}{2} + C = \frac{x^2+5\ln|x^2-1|-1}{2} + C$$

$$= \frac{x^2+5\ln|x^2-1|}{2} + C$$

Answer:

$$\int \frac{\mathrm{x}^3+4\mathrm{x}}{\mathrm{x}^2-1}\,\mathrm{dx} = \frac{x^2+5\ln|x^2-1|}{2} + C$$