Answer on Question #72635 – Math – Statistics and Probability

Question

Suppose that airplane engines operate independently and fail with probability equal to 0.4. Assuming that a plane makes a safe flight if at least one-half of its engines run, determine whether a 4-engine plane or a 2-engine plane has the higher probability for a successful flight.

Solution

Let X be the random variable representing the number of engines running out of n engines in a plane. Let us consider, running of an engine is a success.

Then $q = 0.4, p = 1 - q = 1 - 0.4 = 0.6.$

Trials are independent.

Hence, $X \sim Bin(n, p = 0.6)$

The probability mass function $(p.m.f)$ is

$P(X = x) = b(x; n, 0.6),$ where $x = 0, 1, 2, \dots, n$

$P(X = x) = \binom{n}{x}(0.6)^x(0.4)^{n-x},$ where $x = 0, 1, 2, \dots, n$

a) For the 2-engine plane to make a successful flight, at least one engine must be running.

If $n = 2, X \sim Bin(2, p = 0.6).$

Then

$P(\text{at least one} - \text{half of its engines run}) = P(X \geq 1) = 1 - P(X = 0) =$

$= 1 - \binom{2}{0}(0.6)^0(0.4)^{2-0} = 1 - (0.4)^2 = 0.84$

b) On the other hand, for the 4-engine plane to make a successful flight, at least two engines must be running.

If $n = 4, X \sim Bin(4, p = 0.6).$

Then

$P(\text{at least one} - \text{half of its engines run}) = P(X \geq 2) = 1 - P(X < 2) =$

$= 1 - \big(P(X = 0) + P(X = 1)\big) =$

$= 1 - \left(\binom{4}{0}(0.6)^0(0.4)^{4-0} + \binom{4}{1}(0.6)^1(0.4)^{4-1}\right) =$

$= 1 - ((0.4)^4 + 4(0.6)(0.4)^3) = 0.8208$

Since $0.84 > 0.8208$, the 2-engine plane has a higher probability for a successful flight than the 4-engine plane.

Answer: the 2-engine plane has a higher probability for a successful flight than the 4-engine plane.