Answer on Question #72632, Math / Statistics and Probability

According to a study published by a group of University of Massachusetts sociologists, approximately 60% of the Valium users in the state of Massachusetts first took Valium for psychological problems. Find the probability that among the next 8 users from this state who are interviewed

(a) exactly 3 began taking Valium for psychological problems;

(b) at least 5 began taking Valium for problems that were not psychological.

Solution

Let X be the random variable representing the number of Valium users who began taking Valium for psychological problems out of next 8 users interviewed.

The probability of a success in each trial is $p = 0.6$. Trials are independent.

Hence, X has a binomial distribution with parameters $n = 8$ and $p = 0.6$

$$X \sim Bin(n, p), \quad \text{where } n = 8 \text{ and } p = 0.6$$

The probability mass function $(p.m.f)$ is

$$P(X = x) = b(x; 8, 0.6), \text{where } x = 0, 1, 2, \dots, 8$$

$$P(X = x) = \binom{8}{x}(0.6)^x(1 - 0.6)^{8-x} = \binom{8}{x}(0.6)^x(0.4)^{8-x},$$

where $x = 0, 1, 2, \dots, 8$

(a) Exactly 3 began taking Valium for psychological problems

$$P(X = 3) = \binom{8}{3}(0.6)^3(0.4)^{8-3} = \frac{8!}{3!\,(8-3)!}(0.6)^3(0.4)^5 =$$

$$= \frac{8(7)(6)}{1(2)(3)}(0.6)^3(0.4)^5 = 0.12386304$$

(b) At least 5 began taking Valium for problems that were not psychological

$$P(X \le 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) =$$

$$= \binom{8}{0}(0.6)^0(0.4)^{8-0} + \binom{8}{1}(0.6)^1(0.4)^{8-1} + \binom{8}{2}(0.6)^2(0.4)^{8-2} +$$

$$+0.12386304 = (0.4)^8 + 8(0.6)(0.4)^7 + \frac{8!}{2!\,(8-2)!}(0.6)^2(0.4)^6 +$$

$$+0.12386304 = 0.1736704$$