Answer on Question #72631, Math / Statistics and Probability

It is known that 60% of mice inoculated with a serum are protected from a certain disease. If 5 mice are inoculated, find the probability that

(a) none contracts the disease;

(b) fewer than 2 contract the disease;

(c) more than 3 contract the disease.

Solution

Let X be the random variable representing the number of mice that contract the disease out of 5 mice inoculated with a serum.

If a mouse contracts the disease, we consider that as a success.

If a mouse does not contract the disease, will be a failure.

Then $q = 0.6,\ p = 1 - q = 0.4$. Trials are independent.

Hence, $X \sim Bin(n, p)$, where $n = 5$ and $p = 0.4$.

$P(X = x) = b((x; 5, 0.4) = \binom{5}{x}(0.4)^x(0.6)^{5-x}$, where $x = 0, 1, 2, 3, 4, 5$

(a) none contracts the disease

$$P(X = 0) = \binom{5}{0}(0.4)^0(0.6)^{5-0} = (0.6)^5 = 0.07776$$

(b) fewer than 2 contract the disease

$P(X < 2) = P(X \leq 1) = P(X = 0) + P(X = 1) =$

$= 0.07776 + \binom{5}{1}(0.4)^1(0.6)^{5-1} = 0.07776 + 5(0.4)(0.6)^4 = 0.33696$

(c) more than 3 contract the disease

$P(X > 3) = 1 - P(X \leq 3) =$

$= 1 - \big(P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)\big) =$

$= 1 - (0.33696 + \binom{5}{2}(0.4)^2(0.6)^{5-2} + \binom{5}{3}(0.4)^3(0.6)^{5-3}) =$

$= 1 - (0.33696 + \frac{5!}{2!\,(5-2)!}(0.4)^2(0.6)^3 + \frac{5!}{3!\,(5-3)!}(0.4)^3(0.6)^{5-3}) =$

$= 1 - 0.91296 = 0.08704$

Or

$P(X > 3) = P(X \geq 4) = P(X = 4) + P(X = 5) =$

$= \binom{5}{4}(0.4)^4(0.6)^{5-4} + \binom{5}{5}(0.4)^5(0.6)^{5-5} =$

$= 5(0.4)^4(0.6) + (0.4)^5 = 0.08704$