

Answer on Question #72629, Math / Statistics and Probability

A random committee of size 3 is selected from 4 doctors and 2 nurses. Write a formula for the probability distribution of the random variable X representing the number of doctors on the committee. Find $P(2 \leq X \leq 3)$.

Solution

Let X be the number of doctors on the committee. X is a hypergeometric variable with $X \sim \text{Hypergeometric}(n = 3, N = 6, K = 4)$

$$h(x; N, n, K) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}$$

$$h(x; 6, 3, 4) = \frac{\binom{4}{x} \binom{6-4}{3-x}}{\binom{6}{3}}, \text{ for } x = 1, 2, 3$$

$$P(2 \leq X \leq 3) = h(2; 6, 3, 4) + h(3; 6, 3, 4) = \frac{\binom{4}{2} \binom{6-4}{3-2}}{\binom{6}{3}} + \frac{\binom{4}{3} \binom{6-4}{3-3}}{\binom{6}{3}} =$$

$$= \frac{\frac{4!}{2!(4-2)!} \cdot \frac{2!}{1!(2-1)!}}{6!} + \frac{\frac{4!}{3!(4-3)!} \cdot \frac{2!}{0!(2-0)!}}{6!} =$$

$$= \frac{\frac{4(3)}{1(2)} \cdot 2}{6(5)(4)} + \frac{4 \cdot 1}{6(5)(4)} = \frac{12}{20} + \frac{1}{5} = \frac{4}{5}$$

$$\text{Answer: } h(x; 6, 3, 4) = \frac{\binom{4}{x} \binom{6-4}{3-x}}{\binom{6}{3}}, \text{ for } x = 1, 2, 3; P(2 \leq X \leq 3) = \frac{4}{5}$$