

Answer on Question #72629, Math / Statistics and Probability

A random committee of size 3 is selected from 4 doctors and 2 nurses. Write a formula for the probability distribution of the random variable X representing the number of doctors on the committee. Find $P(2 \leq X \leq 3)$.

Solution

Let X be the number of doctors on the committee. X is a hypergeometric variable with $X \sim \text{Hypergeometric}(n = 3, N = 6, K = 4)$

$$h(x; N, n, K) = \frac{\binom{K}{x} \binom{N - K}{n - x}}{\binom{N}{n}}$$

$$h(x; 6, 3, 4) = \frac{\binom{4}{x} \binom{6 - 4}{3 - x}}{\binom{6}{3}}, \text{ for } x = 1, 2, 3$$

$$P(2 \leq X \leq 3) = h(2; 6, 3, 4) + h(3; 6, 3, 4) = \frac{\binom{4}{2} \binom{6 - 4}{3 - 2}}{\binom{6}{3}} + \frac{\binom{4}{3} \binom{6 - 4}{3 - 3}}{\binom{6}{3}} =$$

$$= \frac{\frac{4!}{2! (4-2)!} \cdot \frac{2!}{1! (2-1)!}}{\frac{6!}{3! (6-3)!}} + \frac{\frac{4!}{3! (4-3)!} \cdot \frac{2!}{0! (2-0)!}}{\frac{6!}{3! (6-3)!}} =$$

$$= \frac{\frac{4(3)}{1(2)} \cdot 2}{\frac{6(5)(4)}{1(2)(3)}} + \frac{4 \cdot 1}{\frac{6(5)(4)}{1(2)(3)}} = \frac{12}{20} + \frac{1}{5} = \frac{4}{5}$$

$$\text{Answer: } h(x; 6, 3, 4) = \frac{\binom{4}{x} \binom{6 - 4}{3 - x}}{\binom{6}{3}}, \text{ for } x = 1, 2, 3; P(2 \leq X \leq 3) = \frac{4}{5}$$

Answer provided by AssignmentExpert.com