Answer on Question #72628, Math / Statistics and Probability

A company is interested in evaluating its current inspection procedure for shipments of 50 identical items. The procedure is to take a sample of 5 and pass the shipment if no more than 2 are found to be defective. What proportion of shipments with 20% defectives will be accepted?

Solution

Let X be the number of defective items in the sample of $n = 5$ taken from $N = 50$ without replacement. Suppose the shipment has M defective items. Thus X is a hypergeometric variable with $X \sim Hypergeometric(n = 5, N = 50, M = pN)$, where p is the proportion of defectives in the shipment. If $p = 0.2$ then $M = 10$.

$$P(X = k) = h_{(M,n,N)}(k) = \frac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}$$

The probability of accepting the shipment is

$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) =$$

$$= h_{(10,5,50)}(0) + h_{(10,5,50)}(1) + h_{(10,5,50)}(2) =$$

$$= \frac{\binom{10}{0}\binom{50-10}{5-0}}{\binom{50}{5}} + \frac{\binom{10}{1}\binom{50-10}{5-1}}{\binom{50}{5}} + \frac{\binom{10}{2}\binom{50-10}{5-2}}{\binom{50}{5}} \approx$$

$$\approx 0.31056 + 0.43134 + 0.20984 \approx 0.9517$$

$$\frac{\binom{10}{0}\binom{50-10}{5-0}}{\binom{50}{5}} = \frac{\frac{10!}{0!\,(10-0)!}\cdot\frac{40!}{5!\,(40-5)!}}{\frac{50!}{5!\,(50-5)!}} = \frac{1\cdot\frac{40(39)(38)(37)(36)}{1(2)(3)(4)(5)}}{\frac{50(49)(48)(47)(46)}{1(2)(3)(4)(5)}} =$$

$$= 0.31056$$

$$\frac{\binom{10}{1}\binom{50-10}{5-1}}{\binom{50}{5}} = \frac{\frac{10!}{1!\,(10-1)!}\cdot\frac{40!}{4!\,(40-4)!}}{\frac{50!}{5!\,(50-5)!}} = \frac{10\cdot\frac{40(39)(38)(37)}{1(2)(3)(4)}}{\frac{50(49)(48)(47)(46)}{1(2)(3)(4)(5)}} =$$

$$= 0.43134$$

$$\frac{\binom{10}{2}\binom{50-10}{5-2}}{\binom{50}{5}} = \frac{\frac{10!}{2!\,(10-2)!}\cdot\frac{40!}{3!\,(40-3)!}}{\frac{50!}{5!\,(50-5)!}} = \frac{\frac{10(9)}{2}\cdot\frac{40(39)(38)}{1(2)(3)}}{\frac{50(49)(48)(47)(46)}{1(2)(3)(4)(5)}} =$$

$$= 0.20984$$

Answer: $\approx 0.9517; 95.17\%$.