Answer on Question #72627, Math / Statistics and Probability

Find the probability of being dealt a bridge hand of 13 cards containing 5 spades, 2 hearts, 3 diamonds, and 3 clubs.

Solution

A bridge deck has 52 cards with 13 cards in each of four suits: spades, hearts, diamonds, and clubs.

Let $X_1, X_2, X_3$ and $X_4$ are the random variables which denote the number of spades, hearts, diamonds, and clubs respectively, in a bridge hand of 13 cards.

Thus, $X_1, X_2, X_3$ and $X_4$ jointly have a multivariate hypergeometric distribution with parameters, $N = 52, n = 13$ and $a_i = 13, \forall i = 1, 2, 3, 4$.

Joint probability mass function of $X_1, X_2, X_3$ and $X_4$ is given by

$$h\left(\begin{matrix} x_1, x_2, x_3, x_4 \\ a_1 = 13, a_2 = 13, a_3 = 13, a_4 = 13 \\ N = 52, n = 13 \end{matrix}\right)$$

$$= \frac{\binom{a_1}{x_1}\binom{a_2}{x_2}\binom{a_3}{x_3}\binom{a_4}{x_4}}{\binom{N}{n}} \text{ with } \sum_{i=1}^{4} x_i = n \text{ and } \sum_{i=1}^{4} a_i = N$$

$$= \frac{\binom{13}{x_1}\binom{13}{x_2}\binom{13}{x_3}\binom{13}{x_4}}{\binom{52}{13}}$$

We have that $x_1 = 5, x_2 = 2, x_3 = 3, x_4 = 3$. Then

$$h\left(\begin{matrix} x_1 = 5, x_2 = 2, x_3 = 3, x_4 = 3 \\ a_1 = 13, a_2 = 13, a_3 = 13, a_4 = 13 \\ N = 52, n = 13 \end{matrix}\right) =$$

$$= \frac{\binom{13}{5}\binom{13}{2}\binom{13}{3}\binom{13}{3}}{\binom{52}{13}} =$$

$$= \frac{\frac{13!}{5!\,(13-5)!} \cdot \frac{13!}{2!\,(13-2)!} \cdot \frac{13!}{3!\,(13-3)!} \cdot \frac{13!}{3!\,(13-3)!}}{\frac{52!}{13!\,(52-13)!}} =$$

$$= \frac{1287(78)(286)(286)}{635013559600} \approx$$

$$\approx 0.01293$$