What is the probability that a waitress will refuse to serve alcoholic beverages to only 2 minors if she randomly checks the IDs of 5 among 9 students, 4 of whom are minors?

Solution

Let X be the random variable which denotes the number of minors among the 5 students selected at random for ID checking.

The total number of students $N = 9$.

The number of students who are not of legal age (minor) $k = 4$.

Hence, X has a hypergeometric distribution with parameters $N = 9, n = 5$ and $k = 4$.

$$X \sim HyperGeom(N, k, n)$$

$p.m.f$ of X is given by

$$h(x; N = 9, n = 5, k = 4) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}},$$

where $max\{0, n - (N - k)\} \le x \le min\{n, k\}$

$i.e.\ 0 \le x \le 4$

The probability that a waitress will refuse to serve alcoholic beverages to only 2 minors

$$P(X = 2) = \frac{\binom{4}{2}\binom{9-4}{5-2}}{\binom{9}{5}} = \frac{\binom{4}{2}\binom{5}{3}}{\binom{9}{5}} = \frac{\frac{4!}{2!(4-2)!} \cdot \frac{5!}{3!(5-3)!}}{\frac{9!}{5!(9-5)!}} =$$

$$= \frac{\frac{4(3)}{1(2)} \cdot \frac{4(5)}{1(2)}}{\frac{9(8)(7)(6)}{1(2)(3)(4)}} = \frac{6(10)}{18(7)} = \frac{10}{21} \approx 0.47619$$

Answer: $\frac{10}{21} \approx 0.47619$.

Step-by-Step Solution:

Step 1 of 3

No. of students who are not of legal age(minor) $= 4$

total no. of students $= 9$

Let X be the random variable which denotes the no. of minorss among the 5

students selected at random for ID checking

$\therefore X$ has a hypergeometric distribution with parameters $N = 9$, $n = 5$ and $k = 4$

and p.m.f of X is given by

$$h(x; N = 9, \ n = 5, k = 4) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$\text{where } \max\{0, n - (N-k)\} \le x \le \min\{n, k\}$$

$$\text{i.e, } 0 \le x \le 4$$