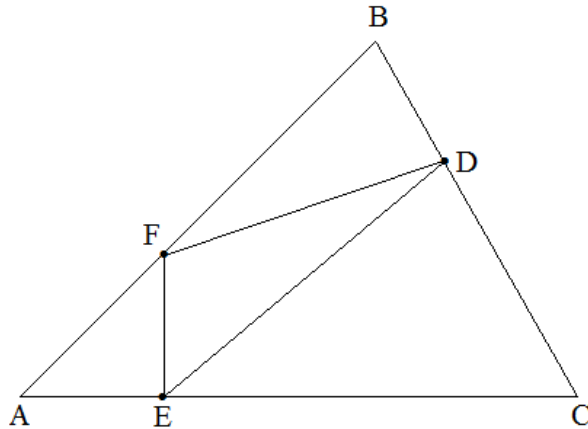


Answer on Question #72600, Math / Geometry

Say in (Triangle) ΔABC D, E, F are three points on BC, CA, AB respectively, such that $AE:EC = 1:3, DC:DB = 2:1, BF:FA = 3:2$.

What is the area of (Triangle) ΔDEF ?

Solution



$$S_{\Delta ABC} = \frac{1}{2}(AB)(AC) \sin A = \frac{1}{2}(AB)(BC) \sin B = \frac{1}{2}(AC)(BC) \sin C$$

$$AE:EC = 1:3 \Rightarrow AE = \frac{1}{4}AC, EC = \frac{3}{4}AC$$

$$DC:DB = 2:1 \Rightarrow DC = \frac{2}{3}BC, DB = \frac{1}{3}BC$$

$$BF:FA = 3:2 \Rightarrow BF = \frac{3}{5}AB, FA = \frac{2}{5}AB$$

$$S_{\Delta ABC} = S_{\Delta AFE} + S_{\Delta FBD} + S_{\Delta DCE} + S_{\Delta DEF}$$

$$S_{\Delta AFE} = \frac{1}{2}(FA)(AE) \sin A = \frac{1}{2}\left(\frac{2}{5}AB\right)\left(\frac{1}{4}AC\right) \sin A = \frac{1}{10}S_{\Delta ABC}$$

$$S_{\Delta FBD} = \frac{1}{2}(BF)(DB) \sin B = \frac{1}{2}\left(\frac{3}{5}AB\right)\left(\frac{1}{3}BC\right) \sin B = \frac{1}{5}S_{\Delta ABC}$$

$$S_{\Delta DCE} = \frac{1}{2}(DC)(EC) \sin C = \frac{1}{2}\left(\frac{2}{3}BC\right)\left(\frac{3}{4}AC\right) \sin C = \frac{1}{2}S_{\Delta ABC}$$

$$S_{\Delta DEF} = S_{\Delta ABC} - \frac{1}{10}S_{\Delta ABC} - \frac{1}{5}S_{\Delta ABC} - \frac{1}{2}S_{\Delta ABC} = \frac{1}{5}S_{\Delta ABC}$$

$$\text{Answer: } S_{\Delta DEF} = \frac{1}{5}S_{\Delta ABC}.$$