Answer on Question #72592, Math / Statistics and Probability

A process yields 10% defective items. If 100 items are randomly selected from the process, what is the probability that the number of defectives.

(a) exceeds 13?

(b) is less than 8?

Solution

Let X be the number of defectives in the 100 items.

Use the Normal Approximation to the Binomial Distribution Theorem.

For large n, X has approximately a normal distribution with $\mu = np$ and $\sigma^2 = np(1-p)$ and

$$P(X < x) \approx P\left(Z < \frac{x - 0.5 - np}{\sqrt{np(1-p)}}\right)$$

$$P(X \le x) \approx P\left(Z < \frac{x + 0.5 - np}{\sqrt{np(1-p)}}\right)$$

$$P(X > x) \approx P\left(Z > \frac{x + 0.5 - np}{\sqrt{np(1-p)}}\right)$$

$$P(X \ge x) \approx P\left(Z > \frac{x - 0.5 - np}{\sqrt{np(1-p)}}\right)$$

We have that

$$p = 0.1, n = 100, np = 100(0.1) = 10, \sqrt{np(1-p)} = \sqrt{100(0.1)(1-0.1)} = 3$$

(a) $P(X > 13) = 1 - P(X \le 13) \approx 1 - P\left(Z \le \frac{13 + 0.5 - 10}{3}\right) =$

$= 1 - P(Z \le 1.17) = 0.5 - 0.3790 = 0.1210$

(b) $P(X < 8) \approx P\left(Z < \frac{8 - 0.5 - 10}{3}\right) = P(Z < -0.83) = 0.2033$