

## Answer to Question #72482, Math / Discrete Mathematics

Solve this degeneracy problem

						Supply
	11	2 (8)	8	6 (6)	2 (4)	18
	9 (10)	9	12	9	6	10
	7	6	3 (8)	7	7	8
	9 (2)	3	5	6 (2)	11	4
Demand	12	8	8	8	4	

### Solution.

We assign artificial quantity  $d$  to the smallest unoccupied cell. The quantity  $d$  is small and it does not affect the supply and demand constraints.

						Supply
	11	2 (8)	8	6 (6)	2 (4)	18
	9 (10)	9	12	9	6	10
	7	6	3 (8)	7	7	8
	9 (2)	3 ( $d$ )	5	6 (2)	11	4
Demand	12	8	8	8	4	

### Stepping Stone Method

Select an unoccupied cell. Beginning at this cell, trace a closed path, starting from the selected unoccupied cell until finally returning to that same unoccupied cell.

Assign plus (+) and minus (-) signs alternatively on each corner cell of the closed path just traced, beginning with the plus sign at unoccupied cell to be evaluated.

Add the unit transportation costs associated with each of the cell traced in the closed path. This will give net change in terms of cost.

Repeat above steps until all unoccupied cells are evaluated.

For cell (1,1) :

	1	2	3	4	5	Supply
1	11 (+)	2 (8)	8	6 (6) (-)	2 (4)	18
2	9 (10)	9	12	9	6	10
3	7	6	3 (8)	7	7	8
4	9 (2) (-)	3 ( $d$ )	5	6 (2) (+)	11	4
Demand	12	8	8	8	4	

## Answer to Question #72482, Math / Discrete Mathematics

The increase in the transportation cost per unit quantity of reallocation:

$$11 - 6 + 6 - 9 = 2$$

For cell (1,3) :

	1	2	3	4	5	Supply
1	11	2 (8)	8 (+)	6 (6) (-)	2 (4)	18
2	9 (10)(+)	9	12(-)	9	6	10
3	7	6	3 (8)	7	7	8
4	9 (2) (-)	3 (d)	5	6 (2) (+)	11	4
Demand	12	8	8	8	4	

The increase in the transportation cost per unit quantity of reallocation:

$$8 - 6 + 6 - 9 + 9 - 12 = -4$$

For cell (2,2) :

	1	2	3	4	5	Supply
1	11	2 (8)(-)	8	6 (6)(+)	2 (4)	18
2	9 (10)(-)	9(+)	12	9	6	10
3	7	6	3 (8)	7	7	8
4	9 (2) (+)	3 (d)	5	6 (2) (-)	11	4
Demand	12	8	8	8	4	

The increase in the transportation cost per unit quantity of reallocation:

$$9 - 2 + 6 - 6 + 9 - 10 = 6$$

For cell (2,3) :

	1	2	3	4	5	Supply
1	11(-)	2 (8)(+)	8	6 (6)	2 (4)	18
2	9 (10)	9(-)	12(+)	9	6	10
3	7	6(+)	3 (8)(-)	7	7	8
4	9 (2) (+)	3 (d)(-)	5	6 (2)	11	4
Demand	12	8	8	8	4	

The increase in the transportation cost per unit quantity of reallocation:

$$12 - 3 + 6 - 3 + 9 - 11 + 2 - 9 = 3$$

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For cell (2,4) :

	1	2	3	4	5	Supply
1	11	2 (8)	8	6 (6)	2 (4)	18
2	9 (10)(-)	9	12	9(+)	6	10
3	7	6	3 (8)	7	7	8
4	9 (2) (+)	3 (d)	5	6 (2)(-)	11	4
Demand	12	8	8	8	4	

The increase in the transportation cost per unit quantity of reallocation:

$$9 - 6 + 9 - 9 = 3$$

For cell (2,5) :

	1	2	3	4	5	Supply
1	11	2 (8)	8	6 (6)(+)	2 (4)(-)	18
2	9 (10)	9	12	9(-)	6(+)	10
3	7	6	3 (8)	7	7	8
4	9 (2)	3 (d)	5	6 (2)	11	4
Demand	12	8	8	8	4	

The increase in the transportation cost per unit quantity of reallocation:

$$6 - 2 + 6 - 9 = 1$$

For cell (3,1) :

	1	2	3	4	5	Supply
1	11(-)	2 (8)	8	6 (6)(+)	2 (4)	18
2	9 (10)	9	12	9	6	10
3	7(+)	6(-)	3 (8)	7	7	8
4	9 (2)	3 (d)(+)	5	6 (2)(-)	11	4
Demand	12	8	8	8	4	

The increase in the transportation cost per unit quantity of reallocation:

$$7 - 11 + 6 - 6 + 3 - 6 = -7$$

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For cell (3,2) :

	1	2	3	4	5	Supply
1	11	2 (8)	8	6 (6)	2 (4)	18
2	9 (10)(+)	9(-)	12	9	6	10
3	7(-)	6(+)	3 (8)	7	7	8
4	9 (2)	3 (d)	5	6 (2)	11	4
Demand	12	8	8	8	4	

The increase in the transportation cost per unit quantity of reallocation:

$$6 - 9 + 9 - 7 = -1$$

For cell (3,4) :

	1	2	3	4	5	Supply
1	11	2 (8)	8	6 (6)	2 (4)	18
2	9 (10)	9	12	9	6	10
3	7	6	3 (8)(-)	7(+)	7	8
4	9 (2)	3 (d)	5(+)	6 (2)(-)	11	4
Demand	12	8	8	8	4	

The increase in the transportation cost per unit quantity of reallocation:

$$7 - 6 + 5 - 3 = 3$$

For cell (3,5) :

	1	2	3	4	5	Supply
1	11	2 (8)	8	6 (6)(+)	2 (4)(-)	18
2	9 (10)	9	12	9	6	10
3	7	6	3 (8)	7(-)	7(+)	8
4	9 (2)	3 (d)	5	6 (2)	11	4
Demand	12	8	8	8	4	

The increase in the transportation cost per unit quantity of reallocation:

$$7 - 2 + 6 - 7 = 4$$

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For cell (4,3) :

	1	2	3	4	5	Supply
1	11	2 (8)	8	6 (6)	2 (4)	18
2	9 (10)	9	12	9	6	10
3	7	6	3 (8)(-)	7(+)	7	8
4	9 (2)	3 (d)	5(+)	6 (2)(-)	11	4
Demand	12	8	8	8	4	

The increase in the transportation cost per unit quantity of reallocation:

$$5 - 6 + 7 - 3 = 3$$

For cell (4,5) :

	1	2	3	4	5	Supply
1	11	2 (8)	8	6 (6)(+)	2 (4)(-)	18
2	9 (10)	9	12	9	6	10
3	7	6	3 (8)	7	7	8
4	9 (2)	3 (d)	5	6 (2)(-)	11(+)	4
Demand	12	8	8	8	4	

The increase in the transportation cost per unit quantity of reallocation:

$$11 - 6 + 6 - 2 = 9$$

The cell (3,1) is having the maximum improvement potential, which is equal to  $-7$ .

The maximum amount that can be allocated to the cell (3,1) is 8.

The improved solution:

	1	2	3	4	5	Supply
1	11	2 (8)	8	6 (6)	2 (4)	18
2	9 (2)	9	12(8)	9	6	10
3	7(8)	6	3	7	7	8
4	9 (2)	3 (d)	5	6 (2)	11	4
Demand	12	8	8	8	4	

The optimal solution:

$$2 \cdot 8 + 6 \cdot 6 + 2 \cdot 4 + 9 \cdot 2 + 12 \cdot 8 + 7 \cdot 8 + 9 \cdot 2 + 6 \cdot 2 = 260$$