

Question 1. Let G be a group and H be a subgroup of G . Define the normalizer of H in G by $N(H) = \{g \in G \mid gHg^{-1} = H\}$. Prove that $N(H)$ is a subgroup of G .

Solution. Obviously, $e \in N(H)$, because $eg = ge = g$ for all elements $g \in G$. Furthermore, if $g_1, g_2 \in N(H)$, then

$$g_1g_2H(g_1g_2)^{-1} = g_1g_2Hg_2^{-1}g_1^{-1} = g_1(g_2Hg_2^{-1})g_1^{-1} = g_1Hg_1^{-1} = H.$$

Finally, prove that $g^{-1} \in N(H)$ for all $g \in N(H)$. We have

$$g^{-1}Hg = g^{-1}(gHg^{-1})g = g^{-1}gHg^{-1}g = H.$$

□