

Question 1. Give an example of a group G and two elements a, b in G such that “the order” $|ab|$ does not equal “the order” $|a||b|$.

Solution. Consider a group G , consisting of the identity e and an element g , such that $g^2 = e$ (it is in fact isomorphic to \mathbb{Z}_2). Take $a = b = g$. Then $|a| = 2$, because $a \neq e$ and $a^2 = e$. Therefore, $|a||b| = 4$. Since $ab = g^2 = e$, we have $|ab| = 1$. Thus, $|ab| \neq |a||b|$. \square