Answer on Question #72485, Math / Geometry

Q. Compute the torsion of

$$\gamma(t) = (\cos^3 t, \sin^3 t)$$

Solution

Torsion: $\tau(t) = \dfrac{(\overrightarrow{\gamma'}(t) \times \overrightarrow{\gamma''}(t)) \cdot \overrightarrow{\gamma'''}(t)}{\left\|\overrightarrow{\gamma'}(t) \times \overrightarrow{\gamma''}(t)\right\|^2}$

$\gamma(t) = (\cos^3 t, \sin^3 t)$

A plane curve with non-vanishing curvature has zero torsion at all points.

$\overrightarrow{\gamma'}(t) = (-3\sin t\cos^2 t, 3\sin^2 t\cos t, 0)$

$\overrightarrow{\gamma''}(t) = (-3\cos^3 t + 6\sin^2 t\cos t, -3\sin^3 t + 6\sin t\cos^2 t, 0)$

$\overrightarrow{\gamma'''}(t) = (21\sin t\cos^2 t - 6\sin^3 t, -21\sin^2 t\cos t + 6\cos^3 t, 0)$

$\overrightarrow{\gamma'}(t) \times \overrightarrow{\gamma''}(t) =$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3\sin t\cos^2 t & 3\sin^2 t\cos t & 0 \\ -3\cos^3 t + 6\sin^2 t\cos t & -3\sin^3 t + 6\sin t\cos^2 t & 0 \end{vmatrix} =$$

$= -9\vec{k}(\sin^4 t\cos^2 t + \sin^2 t\cos^4 t) = -9(\sin^2 t\cos^2 t)\vec{k}$

$\left(\overrightarrow{\gamma'}(t) \times \overrightarrow{\gamma''}(t)\right) \cdot \overrightarrow{\gamma'''}(t) =$

$= (0)(21\sin t\cos^2 t - 6\sin^3 t) + (0)(-21\sin^2 t\cos t + 6\cos^3 t) +$

$+(-9(\sin^2 t\cos^2 t))(0) = 0$

$\left\|\overrightarrow{\gamma'}(t) \times \overrightarrow{\gamma''}(t)\right\|^2 = \left(\sqrt{(0)^2 + (0)^2 + (-9(\sin^2 t\cos^2 t))^2}\right)^2 =$

$= 81\sin^4 t\cos^4 t$

$\tau(t) = 0$