COMPUTE THE TORSION OF $\gamma(t) = (\frac{1}{3}(1+t)^{3/2}, \frac{1}{3}(1-t)^{3/2}, t/\sqrt{2})$
Answer:

$$\frac{1}{4}\frac{\sqrt{2}}{\sqrt{1-t}\sqrt{1+t}}$$

Formula for torsion:

$$\frac{(\gamma', \gamma'', \gamma''')}{|[\gamma', \gamma'']|^2}$$

$a * b$ – dot product. $[a, b]$ – vector product. $|a|$ – vector length. $(a, b, c) := a * [b, c]$ – triple product.

Triple product can be computed as a determinant of a matrix:

$$(a, b, c) = \det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

It is also convenient to remember that triple product is invariant under a circular shift of its three operands (a, b, c):

$$(a, b, c) = a * [b, c] = c * [a, b]$$

Tangent vectors:

$$\gamma' = (\frac{1}{2}\sqrt{1+t}, -\frac{1}{2}\sqrt{(1-t)}, \frac{1}{2}\sqrt{2})$$

$$\gamma'' = (\frac{1}{4\sqrt{1+t}}, \frac{1}{4\sqrt{1-t}}, 0)$$

$$\gamma''' = (-\frac{1}{8(1+t)^{3/2}}, \frac{1}{8(1-t)^{3/2}}, 0)$$

Let's first compute vector product:

$$[\gamma', \gamma''] = \det \begin{bmatrix} i & j & k \\ \frac{1}{2}\sqrt{1+t} & -\frac{1}{2}\sqrt{1-t} & \frac{1}{2}\sqrt{2} \\ \frac{1}{4\sqrt{1+t}} & \frac{1}{4\sqrt{1-t}} & 0 \end{bmatrix} = \frac{j\sqrt{2}\sqrt{1-t} - i\sqrt{2}\sqrt{1+t} + k2}{8\sqrt{1-t}\sqrt{1+t}}$$

The squared length of the vector:

$$|[\gamma', \gamma'']|^2 = \frac{1}{32|1-t|} + \frac{1}{32|1+t|} + \frac{1}{16|1+t||1-t|} =$$

And since there exist natural bounds on t: $-1 < t < 1$:

$$= -\frac{1}{8(t^2-1)}$$

Now triple product is:

$$(\gamma', \gamma'', \gamma''') = [\gamma', \gamma''] * \gamma''' =$$

$$(\frac{1}{8}\frac{-\sqrt{2}}{\sqrt{1-t}}, \frac{1}{8}\frac{\sqrt{2}}{\sqrt{1+t}}, \frac{1}{4}\frac{1}{\sqrt{1-t}\sqrt{1+t}})*(-\frac{1}{8(1+t)^{3/2}}, \frac{1}{8(1-t)^{3/2}}, 0) = \frac{1}{32}\frac{\sqrt{2}}{(1+t)^{3/2}(1-t)^{3/2}}$$

And finally the torsion is:

$$\frac{1}{32}\frac{\sqrt{2}}{(1+t)^{3/2}(1-t)^{3/2}} : -\frac{1}{8(t^2-1)} = \frac{1}{4}\frac{\sqrt{2}}{\sqrt{1-t}\sqrt{1+t}}$$