

Question 72480:

Let S be a nonempty subset of plane \mathbb{R}^2 , it is known that every point (x, y) in S satisfies “if $x > 0$, then $y > 0$ ”. Consider the following properties possibly satisfied by points (x, y) in S :

(I) If $x \leq 0$, then $y \leq 0$.

(II) If $y \leq 0$, then $x \leq 0$.

(III) If $y > 0$, then $x > 0$.

Which of the above properties will have to be satisfied by all points (x, y) in S ?

(a) (II) only

(b) (III) only

(c) (I) and (II)

(d) (I) and (III)

(e) (II) and (III)

Solution:

(I), (III) The point $(-5, 5)$ lies in S . This is a counterexample.

(II) If $y \leq 0$ and x is not less or equal to 0, then $y \leq 0$ and $x > 0$, then $y \leq 0$ and $y > 0$. This is a contradiction.

Answer:

(a) (II) only.