Answer to Question #72474, Math / Discrete Mathematics

Solve the given problem by NWCM (north-west corner method), LCM (the least cost method) and VAM (Vogel's approximation method) and find optimal solution by UV method.

					$Supply$
	11	13	17	14	250
	16	18	10	60	300
	21	24	13	10	400
$Demand$	200	225	250	275	

Solution.

North-west corner method:

					$Supply$
	200	50			250
		175	125		300
			125	275	400
$Demand$	200	225	250	275	

$$Total\ Cost = 200 \cdot 11 + 50 \cdot 13 + 175 \cdot 18 + 125 \cdot 10 + 125 \cdot 13 + 275 \cdot 10 = 11625$$

UV method:

$$c_{ij} = u_i + v_j \ for\ occupied\ cells$$

The reduced costs for unoccupied cells:

$$reduced\ cost = c_{ij} - u_i - v_j$$

$$c_{13} = 17 - 5 = 12;\ c_{14} = 14 - 2 = 12;\ c_{21} = 16 - 5 - 11 = 0$$

$$c_{24} = 60 - 5 - 2 = 53;\ c_{31} = 21 - 8 - 11 = 2;\ c_{32} = 24 - 8 - 13 = 3$$

	$v_1 = 11$	$v_2 = 13$	$v_3 = 5$	$v_4 = 2$	$Supply$
$u_1 = 0$	200	50			250
$u_2 = 5$		175	125		300
$u_3 = 8$			125	275	400
$Demand$	200	225	250	275	

Since all the current reduced costs are non–negative, this is the optimal solution.

$$The\ minimum\ cost = Total\ Cost = 11625$$

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Least cost method:

					$Supply$
	200	50			250
		50	250		300
		125		275	400
$Demand$	200	225	250	275	

$$Total\ Cost = 200 \cdot 11 + 50 \cdot 13 + 50 \cdot 18 + 125 \cdot 24 + 250 \cdot 10 + 275 \cdot 10 = 12000$$

Vogel's approximation method:

	1	2	3	4	$Supply$
1	11	13	17	14	250
2	16 (50)	18	10 (250)	60	300
3	21	24	13	10	400
$Demand$	200	225	250	275	

Penalties:

$$13 - 11 = 2\ for\ row\ 1$$

$$16 - 10 = 6\ for\ row\ 2$$
$$13 - 10 = 3\ for\ row\ 3$$

$$16 - 11 = 5\ for\ column\ 1$$

$$18 - 13 = 5\ for\ column\ 2$$

$$13 - 10 = 3\ for\ column\ 3$$

$$14 - 10 = 4\ for\ column\ 4$$

The highest penalty occurs in the second row. The minimum c_{ij} in this row is $c_{23} = 10$. So $x_{23} = 300$ and second row is eliminated.

	1	2	3	4	$Supply$
1	11	13 (225)	17	14	250
3	21	24	13	10	400
$Demand$	150	225	0	275	

Penalties:

$$13 - 11 = 2\ for\ row\ 1$$

$$13 - 10 = 3 \; for \; row \; 3$$

$$21 - 11 = 10 \; for \; column \; 1$$

$$24 - 13 = 11 \; for \; column \; 2$$

$$17 - 13 = 4 \; for \; column \; 3$$

$$14 - 10 = 4 \; for \; column \; 4$$

The highest penalty occurs in the second column. The minimum c_{ij} in this column is $c_{12} = 13$. So $x_{12} = 300$ and second column is eliminated.

	1	3	4	$Supply$
1	11 (25)	17	14	25
3	21 (125)	13	10	400
$Demand$	150	0	275	

Penalties:

$$14 - 11 = 3 \; for \; row \; 1$$

$$13 - 10 = 3 \; for \; row \; 3$$

$$21 - 11 = 10 \; for \; column \; 1$$

$$17 - 13 = 4 \; for \; column \; 3$$

$$14 - 10 = 4 \; for \; column \; 4$$

The highest penalty occurs in the first column. The minimum c_{ij} in this column is $c_{11} = 11$. So $x_{11} = 150$ and first column is eliminated.

	3	4	$Supply$
1	17	14	0
3	13	10 (275)	275
$Demand$	0	275	

Penalties:

$$17 - 14 = 3 \; for \; row \; 1$$

$$13 - 10 = 3 \; for \; row \; 3$$

$$17 - 13 = 4 \; for\ column\ 3$$

$$14 - 10 = 4 \; for\ column\ 4$$

The highest penalty occurs in the 4th column. The minimum c_{ij} in this column is $c_{34} = 10$. So $x_{34} = 275$ and 4th column is eliminated.

$$Total\ Cost = 16 \cdot 50 + 10 \cdot 250 + 13 \cdot 225 + 11 \cdot 25 + 21 \cdot 125 + 10 \cdot 275 = 11875$$