Answer on Question #72428 – Math – Statistics and Probability

Question

Suppose that Peter visits the hostel mess sometime between 12:00 noon and 2:00 p.m. He leaves the mess after finishing his lunch in 20 minutes. Likewise, Gwen visits the same hostel mess sometime between 1:00 p.m. and 2:00 p.m.. She also leaves the mess after finishing her lunch in 20 minutes. If the probability of arrival of each person is uniform across the given intervals respectively, find the probability that they meet in the mess.

Solution

Let x be the time to come to the hostel mess of Peter and y the time of arrival in the hostel mess of Gwen. Since Peter visits the hostel mess sometime between 12:00 noon and 2:00 p.m. and Gwen visits the same hostel mess sometime between 1:00 p.m. and 2:00 p.m., we can assume that $0 \le x \le 120$ and $60 \le y \le 120$. So, the sample space of an experiment is $\Omega = \{(x, y) | 0 \le x \le 120, 60 \le y \le 120\}$.

Each pair (x, y) is the result of the experiment. In order for Peter and Gwen to meet, it's enough to have $|x-y| \le 20$. So, the event that Peter and Gwen will meet is $A = \{(x, y) \in \Omega | | x - y | \le 20\}$.



So,

$$P(A) = \frac{S(A)}{S(\Omega)} = \frac{60 \cdot 120 - \left(60 \cdot 40 + \frac{1}{2} \cdot 60 \cdot 60 + \frac{1}{2} \cdot 40 \cdot 40\right)}{60 \cdot 120} = \frac{60 \cdot 120 - \left(60 \cdot 40 + 30 \cdot 60 + 20 \cdot 40\right)}{60 \cdot 120} = \frac{6 \cdot 12 - \left(6 \cdot 4 + 3 \cdot 6 + 2 \cdot 4\right)}{6 \cdot 12} = \frac{36 - 12 - 9 - 4}{6 \cdot 6} = \frac{11}{36} \approx 0.3056.$$

Answer: the probability that Peter and Gwen meet in the mess is 0.3056.

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