Answer on Question #72400 – Math – Geometry

Question

UM=2x+14

MS=14+x

Find US

Solution

In the common case, the given points *U*, *M* and *S* (the end points of *UM*, *MS* and *US*) form a triangle *UMS* (see Fig. 1). We can apply the cosine rule to find unknown side *US*:

$$US^2 = UM^2 + MS^2 - 2UM \cdot MS \cos \alpha, \qquad (1)$$

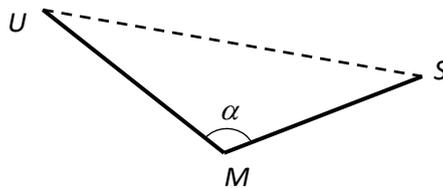where α denotes the angle between *UM* and *MS*.

Fig. 1. A triangle *UMS*.

Hence

$$US^2 = (2x + 14)^2 + (14 + x)^2 - 2(2x + 14)(14 + x)\cos\alpha,$$

$$US^2 = (5x^2 - 4x^2 \cos\alpha) + (84x + 392)(1 - \cos\alpha),$$

$$US = \sqrt{(5x^2 - 4x^2 \cos\alpha) + (84x + 392)(1 - \cos\alpha)}. \qquad (2)$$

In particular cases, when $\alpha = 0^\circ$ or $\alpha = 180^\circ$, the triangle *UMS* degenerates (see Fig. 2) and equation (2) can be simplified:

$$US = x, \; if \; \alpha = 0^\circ; \qquad (3)$$

$$US = 3x + 28, \; if \; \alpha = 180^\circ. \qquad (4)$$

Fig. 2. The cases of a degenerated triangle *UMS*.

Answer provided by https://www.AssignmentExpert.com