

**Question:** If  $270^\circ < m^\circ < 360^\circ$ , and  $\sqrt{3}\sin m^\circ + \cos m^\circ = 2\sin 2011^\circ$ . then  $m=?$

Solution:

$$\sqrt{3}\sin m^\circ + \cos m^\circ = 2 \sin 2011^\circ \quad (1)$$

Let's divide both sides of the equation (1) by 2:

$$\frac{\sqrt{3}}{2} \sin m^\circ + \frac{1}{2} \cos m^\circ = \sin 2011^\circ$$

Let's substitute  $\frac{\sqrt{3}}{2}$  with  $\cos 30^\circ$  and  $\frac{1}{2}$  with  $\sin 30^\circ$ :

$$\cos 30^\circ \sin m^\circ + \sin 30^\circ \cos m^\circ = \sin 2011^\circ$$

Left side of the equation is sinus of the sum of  $30^\circ$  and  $m^\circ$ .

$$\sin(m+30^\circ) = \sin 2011^\circ$$

$$\sin(m+30^\circ) - \sin 2011^\circ = 0$$

Here let's use the formula for the subtraction of sinuses:  $\sin a - \sin b = 2 \sin \frac{a-b}{2} \cos \frac{a+b}{2}$

$$2 \sin \frac{m+30-2011}{2} \cos \frac{m+30+2011}{2} = 0$$

$$\sin \frac{m-1981}{2} \cos \frac{m+2041}{2} = 0 \quad (2)$$

From equation (2) we get these solutions:  $\sin \frac{m-1981}{2} = 0$  or  $\cos \frac{m+2041}{2} = 0$ .

$$1) \sin \frac{m-1981}{2} = 0$$

$$\frac{m-1981}{2} = \pi n$$

$$m-1981 = 2\pi n$$

$$m = 1981 + 2\pi n \quad (3)$$

Here  $1981 = 181^\circ + 5 \cdot 2\pi n$ , so the equation (3) becomes:

$$m = 181^\circ + 2\pi n, \text{ where } n \text{ is an integer.}$$

This solution does not correspond to the condition that  $270^\circ < m^\circ < 360^\circ$ .

$$2) \cos \frac{m+2041}{2} = 0$$

$$\frac{m+2041}{2} = \frac{\pi}{2} + \pi k$$

$$m + 2041 = \pi + 2\pi k \quad (4)$$

As  $2041 = 61^\circ + \pi + 5 \cdot 2\pi k$ , the equation (4) becomes:

$$m = -61^\circ + 2\pi k, \text{ where } k \text{ is an integer.}$$

In the last equation substituting  $k=1$  we get a solution  $m=299^\circ$  that corresponds to the condition that  $270^\circ < m^\circ < 360^\circ$ .

Answer:  $m=299^\circ$