Question:

As far as I can understand, the question was about simplifying the given equation for further solving.

Given equation is:

$$\frac{\sin^8 y}{a^3} + \frac{\cos^8 y}{b^3} = \frac{1}{(a+b)^3}. \qquad (1)$$

Answer:

The first step we have to do is applying the power-reduction formula:

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2},$$

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}.$$

In our case:

$$\sin^8 y = (\sin^2 y)^4 = \left(\frac{1 - \cos 2y}{2}\right)^4,$$

$$\cos^8 y = (\cos^2 y)^4 = \left(\frac{1 + \cos 2\alpha}{2}\right)^4.$$

On the next step, we change the variable. Let:

$$z = \frac{1 - \cos 2y}{2}.$$

Now, the given equation turns to:

$$\frac{z^4}{a^3} + \frac{(1-z)^4}{b^3} = \frac{1}{(a+b)^3},$$

which can be transformed to a quartic equation:

$$z^4 - z^3 \frac{4a^3}{a^3 + b^3} + z^2 \frac{6a^3}{a^3 + b^3} - z \frac{4a^3}{a^3 + b^3} + \frac{a^3}{a^3 + b^3} - \frac{a^3 b^3}{(a+b)^3} = 0,$$

or:

$$z^4 - z^3 \frac{4}{1+c^3} + z^2 \frac{6}{1+c^3} - z \frac{4}{1+c^3} + \frac{1}{1+c^3} - \frac{b^3}{(1+c)^3} = 0, \qquad (2)$$

where $c = b / a$.

Equation (2) can be solved as a quartic equation.