**Question**

How many solutions does the equation

$$\sin(x)\sin(2x)\sin(3x)\dots\sin(11x)\sin(12x) = 0$$

have in the interval $(0, \pi]$ ?

**Solution**

The equation $\sin(12x) = 0$ has solutions in the interval $(0, \pi]$

$$x_1 = \frac{\pi}{12}, x_2 = \frac{\pi}{6}, x_3 = \frac{\pi}{4}, x_4 = \frac{\pi}{3}, x_5 = \frac{5\pi}{12}, x_6 = \frac{\pi}{2}, x_7 = \frac{7\pi}{12}, x_8 = \frac{2\pi}{3},$$

$$x_9 = \frac{3\pi}{4}, x_{10} = \frac{5\pi}{6}, x_{11} = \frac{11\pi}{12}, x_{12} = \pi$$

The equation $\sin(11x) = 0$ has solutions in the interval $(0, \pi]$

$$x_1 = \frac{\pi}{11}, x_2 = \frac{2\pi}{11}, x_3 = \frac{3\pi}{11}, x_4 = \frac{4\pi}{11}, x_5 = \frac{5\pi}{11}, x_6 = \frac{6\pi}{11}, x_7 = \frac{7\pi}{11}, x_8 = \frac{8\pi}{11},$$

$$x_9 = \frac{9\pi}{11}, x_{10} = \frac{10\pi}{11}, x_{11} = \pi$$

The equation $\sin(10x) = 0$ has solutions in the interval $(0, \pi]$

$$x_1 = \frac{\pi}{10}, x_2 = \frac{\pi}{5}, x_3 = \frac{3\pi}{10}, x_4 = \frac{2\pi}{5}, x_5 = \frac{\pi}{2}, x_6 = \frac{3\pi}{5}, x_7 = \frac{7\pi}{10}, x_8 = \frac{4\pi}{5},$$

$$x_9 = \frac{9\pi}{10}, x_{10} = \pi$$

The equation $\sin(9x) = 0$ has solutions in the interval $(0, \pi]$

$$x_1 = \frac{\pi}{9}, x_2 = \frac{2\pi}{9}, x_3 = \frac{\pi}{3}, x_4 = \frac{4\pi}{9}, x_5 = \frac{5\pi}{9}, x_6 = \frac{2\pi}{3}, x_7 = \frac{7\pi}{9}, x_8 = \frac{8\pi}{9},$$

$$x_9 = \pi$$

The equation $\sin(8x) = 0$ has solutions in the interval $(0, \pi]$

$$x_1 = \frac{\pi}{8}, x_2 = \frac{\pi}{4}, x_3 = \frac{3\pi}{8}, x_4 = \frac{\pi}{2}, x_5 = \frac{5\pi}{8}, x_6 = \frac{3\pi}{4}, x_7 = \frac{7\pi}{8}, x_8 = \pi$$

The equation $\sin(7x) = 0$ has solutions in the interval $(0, \pi]$

$$x_1 = \frac{\pi}{7}, x_2 = \frac{2\pi}{7}, x_3 = \frac{3\pi}{7}, x_4 = \frac{4\pi}{7}, x_5 = \frac{5\pi}{7}, x_6 = \frac{6\pi}{7}, x_7 = \pi$$

Therefore, the equation $\sin(x)\sin(2x)\sin(3x)\ldots\sin(11x)\sin(12x) = 0$ has solutions in the interval $(0, \pi\,]$

$$x_1 = \frac{\pi}{12}, x_2 = \frac{\pi}{11}, x_3 = \frac{\pi}{10}, x_4 = \frac{\pi}{9}, x_5 = \frac{\pi}{8}, x_6 = \frac{\pi}{7}, x_7 = \frac{\pi}{6}, x_8 = \frac{2\pi}{11},$$

$$x_9 = \frac{\pi}{5}, x_{10} = \frac{2\pi}{9}, x_{11} = \frac{\pi}{4}, x_{12} = \frac{2\pi}{7}, x_{13} = \frac{3\pi}{11}, x_{14} = \frac{3\pi}{10}, x_{15} = \frac{\pi}{3}, x_{16} = \frac{3\pi}{8},$$

$$x_{17} = \frac{3\pi}{7}, x_{18} = \frac{4\pi}{11}, x_{19} = \frac{2\pi}{5}, x_{20} = \frac{4\pi}{9}, x_{21} = \frac{\pi}{2}, x_{22} = \frac{4\pi}{7}, x_{23} = \frac{5\pi}{12},$$

$$x_{24} = \frac{5\pi}{11}, x_{25} = \frac{5\pi}{9}, x_{26} = \frac{5\pi}{8}, x_{27} = \frac{5\pi}{7}, x_{28} = \frac{6\pi}{11}, x_{29} = \frac{3\pi}{5}, x_{30} = \frac{2\pi}{3},$$

$$x_{31} = \frac{3\pi}{4}, x_{32} = \frac{6\pi}{7}, x_{33} = \frac{7\pi}{12}, x_{34} = \frac{7\pi}{11}, x_{35} = \frac{7\pi}{10}, x_{36} = \frac{7\pi}{9}, x_{37} = \frac{7\pi}{8},$$

$$x_{38} = \frac{8\pi}{11}, x_{39} = \frac{4\pi}{5}, x_{40} = \frac{8\pi}{9}, x_{41} = \frac{9\pi}{11}, x_{42} = \frac{9\pi}{10}, x_{43} = \frac{5\pi}{6}, x_{44} = \frac{10\pi}{11},$$

$$x_{45} = \frac{11\pi}{12}, x_{46} = \pi.$$

There are 46 solutions.