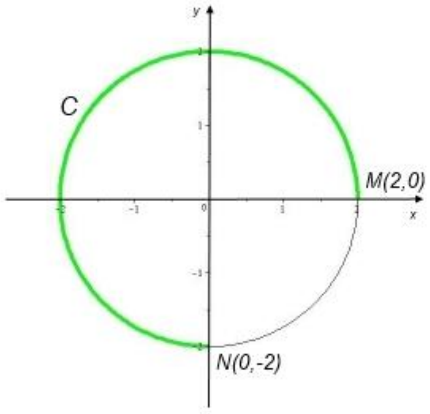
Answer on Question #72171 – Math – Calculus

Question

Calculate the work done by a force F vector = (x − y)i cap + xy j cap in moving a particle counter- clockwise along the circle x2 + y2 = 4 from the point (2, 0) to the point (0, −2)

Solution

Force $\vec{F} = (x-y)\vec{\imath} + xy\vec{\jmath};$
curve $x^2 + y^2 = 4;$
points $M(2,0); N(0.-2).$



The line integral gives the work done by a force $\vec{F}$ on a particle that moving along a curve C:

$$W = \int_C \vec{F}d\vec{l} = \int_C F_x\,dx + F_y dy;$$

$$F_x = x - y; F_y = xy;$$

Let us use the parametrization of the circle, where the radius $r = 2$ is constant:

$$\begin{cases} x = 2\cos t; \\ y = 2\sin t; \end{cases} \begin{cases} dx = -2\sin t dt; \\ dy = 2\cos t dt; \end{cases} \begin{cases} F_x = 2\cos t - 2\sin t; \\ F_y = 4\cos t \sin t; \end{cases} \begin{cases} t_1 = 0 \to x = 2, y = 0 \to M(2,0); \\ t_2 = {}^{3\pi}/_2 \to x = 0, y = -2 \to N(0,-2). \end{cases}$$

$$W = \int_0^{3\pi/2}\{(2\cos t - 2\sin t)\cdot(-2\sin t) + 4\cos t\sin t \cdot 2\cos t\}\,dt = -4\int_0^{3\pi/2}\cos t\sin t\,dt +$$

$$4\int_0^{3\pi/2}(\sin t)^2\,dt + 8\int_0^{3\pi/2}(\cos t)^2\sin t\,dt = -4\int_0^{3\pi/2}\sin t\,d(\sin t) + 2\int_0^{3\pi/2}(1-\cos 2t)dt -$$

$$8\int_0^{3\pi/2}(\cos t)^2 d(\cos t) = -4\frac{(\sin t)^2}{2}\Big|_0^{3\pi/2} + 2\frac{t}{1}\Big|_0^{3\pi/2} - 2\frac{\sin 2t}{2}\Big|_0^{3\pi/2} - 8\frac{(\cos t)^3}{3}\Big|_0^{3\pi/2} = -4\cdot\left(\frac{(-1)^2}{2} - 0\right) +$$

$$2\cdot\left({}^{3\pi}/_2 - 0\right) - (0-0) - 8\cdot\left(0 - \frac{1}{3}\right) = -2 + \frac{8}{3} + 3\pi = \frac{2}{3} + 3\pi.$$

Answer: $W = \frac{2}{3} + 3\pi.$

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