Answer on Question # 72163 – Math – Differential Equations

Question

Obtain all the first and second order partial derivatives of the function: f (x, y)=x^2sin y+y^2cos x

Solution

We have the function of two variables

$$f(x,y) = x^2 \sin y + y^2 \cos x$$

1) Obtain the first order partial derivatives.

Holding y constant and differentiating f with respect to x, we get f_x:

$$f_x = (x^2 \sin y + y^2 \cos x)_x = 2x \sin y - y^2 \sin x$$

Holding x constant and differentiating f with respect to y, we get f_y:

$$f_y = (x^2 \sin y + y^2 \cos x)_y = x^2 \cos y + 2y \cos x$$

2) Obtain the second order partial derivatives.

Holding y constant and differentiating f_x with respect to x, we get f_{xx}:

$$f_{xx} = (f_x)_x = (2x \sin y - y^2 \sin x)_x = 2 \sin y - y^2 \cos x$$

Holding x constant and differentiating f_x with respect to y, we get f_{xy}:

$$f_{xy} = (f_x)_y = (2x \sin y - y^2 \sin x)_y = 2x \cos y - 2y \sin x$$

Holding y constant and differentiating f_y with respect to x, we get f_{yx}:

$$f_{yx} = (f_y)_x = (x^2 \cos y + 2y \cos x)_x = 2x \cos y - 2y \sin x$$

Thus $f_{xy} = f_{yx}$

Holding x constant and differentiating f_y with respect to y, we get f_{yy}:

$$f_{yy} = (f_y)_y = (x^2 \cos y + 2y \cos x)_y = -x^2 \sin y + 2 \cos x$$

Answer:

The first order partial derivatives are

$$f_x = 2x \sin y - y^2 \sin x$$
$$f_y = x^2 \cos y + 2y \cos x$$

The second order partial derivatives are

$$f_{xx} = 2 \sin y - y^2 \cos x$$
$$f_{xy} = f_{yx} = 2x \cos y - 2y \sin x$$
$$f_{yy} = -x^2 \sin y + 2 \cos x$$