

Answer on Question #72162 – Math – Calculus

Question

Obtain the directional derivative for a scalar field $\varphi(x, y, z) = 3xy - yz$ at the point $(1, -2, -1)$ in the direction to the point $(2, 3, 2)$

Solution

The directional derivative is

$$\frac{\partial \varphi}{\partial l} = \frac{\partial \varphi}{\partial x} \cos \alpha + \frac{\partial \varphi}{\partial y} \cos \beta + \frac{\partial \varphi}{\partial z} \cos \gamma,$$

where

$$\cos \alpha = \frac{l_x}{|\vec{l}|}; \cos \beta = \frac{l_y}{|\vec{l}|}; \cos \gamma = \frac{l_z}{|\vec{l}|};$$

$$|\vec{l}| = \sqrt{l_x^2 + l_y^2 + l_z^2}.$$

In our case:

$$\vec{l} = (2 - 1, 3 - (-2), 2 - (-1)) = (1, 5, 3),$$

$$|\vec{l}| = \sqrt{1^2 + 5^2 + 3^2} = \sqrt{35}$$

$$\cos \alpha = \frac{1}{\sqrt{35}}; \cos \beta = \frac{5}{\sqrt{35}}; \cos \gamma = \frac{3}{\sqrt{35}}$$

$$\frac{\partial \varphi}{\partial x} = 3y; \frac{\partial \varphi}{\partial y} = 3x - z; \frac{\partial \varphi}{\partial z} = -y$$

At the point $(1, -2, -1)$:

$$\frac{\partial \varphi}{\partial x} = 3 \cdot (-2) = -6; \frac{\partial \varphi}{\partial y} = 3 \cdot 1 - (-1) = 4; \frac{\partial \varphi}{\partial z} = -(-2) = 2$$

Answer:

$$\frac{\partial \varphi}{\partial l} = -6 \cdot \frac{1}{\sqrt{35}} + 4 \cdot \frac{5}{\sqrt{35}} + 2 \cdot \frac{3}{\sqrt{35}} = \frac{20}{\sqrt{35}} = \frac{4\sqrt{35}}{7}$$