Question 72127:

Two points D and E are taken inside the triangle ABC such as $\angle ABD = \angle EBC$.
From the point D two perpendicular lines DF and DG are drawn to AB and BC respectively.
From the point E two perpendicular lines EH and EI are drawn to AB and BC respectively.
$DF = 7, DG = 8, EI = 15, EH = ?$

Answer:

Let E' be a point on the line BD with perpendicular lines $E'H'$ and $E'I'$ drawn to AB and BC respectively, such that:

$$E'H' = EI \text{ and } E'I' = EH$$

The second equality results from the facts that $\Delta\, E'H'B = \Delta\, EIB$ and $\Delta\, E'I'B = \Delta\, EHB$.

Since $\Delta\, E'H'B \approx \Delta\, DFB$ and $\Delta\, E'I'B \approx \Delta\, DGB$, we have that:

$$EH = E'I' = DG \cdot E'H'/DF = DG \cdot EI/DF = 8 \cdot 15/7 = 17.14.$$

Answer:

$EH = 17.14.$