Question #72105, Math / Geometry

Compute the torsion of the circular helix γ(t)=(acos t, asin t, bt)

Answer.

Arc length parametrization: $y = \frac{1}{c}\left(acos\frac{s}{c}, asin\frac{s}{c}, b\frac{s}{c}\right), where\ c = \sqrt{a^2 + b^2}.$

$y'(t) = \left(-\frac{a}{c}sin\frac{s}{c}, \frac{a}{c}cos\frac{s}{c}, \frac{b}{c}\right).$

$y''(t) = \left(-\frac{a}{c^2}cos\frac{s}{c}, -\frac{a}{c^2}sin\frac{s}{c}, 0\right).$

$y'''(t) = \left(\frac{a}{c^3}sin\frac{s}{c}, -\frac{a}{c^3}cos\frac{s}{c}, 0\right).$

$Torsion\ \tau = \frac{(u'u''u''')}{u''\cdot u''}.$

$$(y'y''y''') = \begin{vmatrix} -\frac{a}{c}sin\frac{s}{c} & \frac{a}{c}cos\frac{s}{c} & \frac{b}{c} \\ -\frac{a}{c^2}cos\frac{s}{c} & -\frac{a}{c^2}sin\frac{s}{c} & 0 \\ \frac{a}{c^3}sin\frac{s}{c} & -\frac{a}{c^3}cos\frac{s}{c} & 0 \end{vmatrix} = \frac{b}{c}\begin{vmatrix} -\frac{a}{c^2}cos\frac{s}{c} & -\frac{a}{c^2}sin\frac{s}{c} \\ \frac{a}{c^3}cos\frac{s}{c} & -\frac{a}{c^3}sin\frac{s}{c} \end{vmatrix} = \frac{a^2b}{c^6}.$$

$$y''\cdot y'' = \left(-\frac{a}{c^2}cos\frac{s}{c}, -\frac{a}{c^2}sin\frac{s}{c}, 0\right)\cdot\left(-\frac{a}{c^2}cos\frac{s}{c}, -\frac{a}{c^2}sin\frac{s}{c}, 0\right) = \frac{a^2}{c^4}.$$

$So\ \tau = \frac{a^2b}{c^6} * \frac{c^4}{a^2} = \frac{b}{c^2} = \frac{b}{a^2+b^2}.$

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