Question #72095, Math / Complex Analysis

Verify Cauchy Goursat formula where $f(z) = z + 1$ and D is the region bounded by the triangle with vertices at 0,3 and $3 + 2i$.

Solution.

Cauchy Goursat formula:

$$\oint_D f(z)dz = 0$$

Then:

$$f(z) = u(x, y) + iv(x, y)$$

$$\oint_D f(z)dz = \oint_D u(x, y)dx - v(x, y)dy + i\oint_D v(x, y)dx + u(x, y)dy$$

In our case:

$$u(x, y) = x + 1\,;\ v(x, y) = y$$

From vertex 0 to vertex 3:

$$y = 0; 0 \le x \le 3; dy = 0$$

$$\int_{0,3} zdz = \int_0^3 (x + 1)dx = \left(\frac{x^2}{2} + x\right)\Big|_0^3 = \frac{9}{2} + 3 = 7.5$$

From vertex 3 to vertex $3 + 2i$:

$$x = 3; 0 \le y \le 2; dx = 0$$

$$\int_{3,3+2i} zdz = -\int_0^2 ydy + i\int_0^2 (3 + 1)dy = -\frac{y^2}{2}\Big|_0^2 + i(4y)_0^2 = -2 + 8i$$

From vertex $3 + 2i$ to vertex 0:

$$y = \frac{2}{3}x;\ 0 \le x \le 3; dy = \frac{2}{3}dx$$

$$\int_{3+2i,0} zdz = \int_3^0 \left(x + 1 - \frac{2}{3}\cdot\frac{2}{3}x\right)dx + i\int_3^0 \left(\frac{2}{3}x + \frac{2}{3}(x+1)\right)dx =$$

$$= \left(\frac{5}{9}\cdot\frac{x^2}{2} + x\right)\Bigg|_3^0 + i\left(\frac{4}{3}\cdot\frac{x^2}{2} + \frac{2}{3}x\right)\Bigg|_3^0 = -\left(\frac{5}{9}\cdot\frac{9}{2} + 3\right) - i\left(\frac{4}{3}\cdot\frac{9}{2} + \frac{2}{3}\cdot 3\right) = -5.5 - 8i$$

So far:

$$\oint_D f(z)dz = \int_{0,3} zdz + \int_{3,3+2i} zdz + \int_{3+2i,0} zdz =$$

$$= 7.5 + (-2 + 8i) + (-5.5 - 8i) = 0$$

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