Question #71945 - Math - Differential Equations

((D^2)-(2DD')+(D'^2))z=(e^(x+y))+(2(x^2)y)

Solution:

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+y} + 2x^2 y$$

$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2}$ parabolic partial differential equation ($B^2 - AC = 0$ where B=-1 and A=C=1).

Change (x,y) to (α,β) $\alpha = \alpha(x,y)$ $\beta = \beta(x,y)$ $\frac{D(\alpha,\beta)}{D(x,y)} = \begin{vmatrix} \frac{\partial \alpha}{\partial x} & \frac{\partial \alpha}{\partial y} \\ \frac{\partial \beta}{\partial x} & \frac{\partial \beta}{\partial y} \end{vmatrix} \neq 0$

Then equation $A\frac{\partial^2 z}{\partial x^2} + 2B\frac{\partial^2 z}{\partial x \partial y} + C\frac{\partial^2 z}{\partial y^2} = F(x,y)$ transform to $A_1\frac{\partial^2 z}{\partial \alpha^2} + 2B_1\frac{\partial^2 z}{\partial \alpha \partial \beta} + C_1\frac{\partial^2 z}{\partial \beta^2} = F_1(\alpha,\beta)$ where

$$\begin{cases} A_1(\alpha,\beta) = A\frac{\partial^2 \alpha}{\partial x^2} + 2B\frac{\partial^2 \alpha}{\partial x \partial y} + C\frac{\partial^2 \alpha}{\partial y^2} \\ B_1(\alpha,\beta) = A\frac{\partial \alpha}{\partial x}\frac{\partial \beta}{\partial x} + B\left(\frac{\partial \alpha}{\partial x}\frac{\partial \beta}{\partial y} + \frac{\partial \alpha}{\partial y}\frac{\partial \beta}{\partial x}\right) + C\frac{\partial \alpha}{\partial y}\frac{\partial \beta}{\partial y} \\ C_1(\alpha,\beta) = A\frac{\partial^2 \beta}{\partial x^2} + 2B\frac{\partial^2 \beta}{\partial x \partial y} + C\frac{\partial^2 \beta}{\partial y^2} \end{cases}$$

If control $B^2 - AC = (B_1{}^2 - A_1C_1)\left(\frac{\partial \alpha}{\partial x}\frac{\partial \beta}{\partial y} - \frac{\partial \alpha}{\partial y}\frac{\partial \beta}{\partial x}\right)^2$

$B_1 = A_1 = 0$ then equation

$$A\frac{\partial^2 \alpha}{\partial x^2} + 2B\frac{\partial^2 \alpha}{\partial x \partial y} + C\frac{\partial^2 \alpha}{\partial y^2} = 0$$

have solution $\alpha = x + y$

Let $\beta = x^2$ (β – any function (x,y) with $\frac{D(\alpha,\beta)}{D(x,y)} = \begin{vmatrix} \frac{\partial \alpha}{\partial x} & \frac{\partial \alpha}{\partial y} \\ \frac{\partial \beta}{\partial x} & \frac{\partial \beta}{\partial y} \end{vmatrix} \neq 0$)

$$\frac{D(\alpha,\beta)}{D(x,y)} = \frac{\partial \alpha}{\partial x}\frac{\partial \beta}{\partial y} - \frac{\partial \alpha}{\partial y}\frac{\partial \beta}{\partial x} = 0 - 2x$$

Equation $A_1 \frac{\partial^2 z}{\partial \alpha^2} + 2B_1 \frac{\partial^2 z}{\partial \alpha \partial \beta} + C_1 \frac{\partial^2 z}{\partial \beta^2} = F_1(\alpha, \beta)$ transform to

$$\frac{\partial^2 z}{\partial \beta^2} = \frac{F_1(\alpha, \beta)}{C_1}$$

$$F_1(\alpha, \beta) = \mathrm{e}^{\alpha} + 2\beta(\alpha - \sqrt{\beta})$$

$$C_1 = 2$$

Solve $\frac{\partial^2 z}{\partial \beta^2} = \frac{\mathrm{e}^{\alpha}}{2} + \beta\left(\alpha - \sqrt{\beta}\right)$

$$z(\beta) = Const_1 + Const_2\beta - \frac{4\beta^{\frac{7}{2}}}{35} + \frac{\alpha\beta^3}{6} + \frac{\beta^2 e^{\alpha}}{2}$$

$$z(x, y) = Const_1 + Const_2 x^2 - \frac{4x^7}{35} + \frac{(x+y)x^6}{6} + \frac{x^4 e^{x+y}}{2}$$

Answer:

$$z(x, y) = Const_1 + Const_2 x^2 - \frac{4x^7}{35} + \frac{(x+y)x^6}{6} + \frac{x^4 e^{x+y}}{2}$$