Answer on Question # 71937, Math / Differential Equations

Question

d^2x/dt^2 -6dx/dt +9x = 0, x(0) = 6, x(0) = -1

Solution. We have the initial value problem

$$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 9x = 0, \quad x(0) = 6, \quad x(0) = -1$$

First we find the general solution of this differential equation.

Seek a solution in the form $x = e^{mt}$. After substitution of $\frac{dx}{dt} = me^{mt}$ and $\frac{d^2x}{dt^2} = m^2e^{mt}$, equation becomes

$$m^2e^{mt} - 6me^{mt} + 9e^{mt} = 0$$

or

$$e^{mt}(m^2 - 6m + 9) = 0.$$

Since $e^{mt} \neq 0$ the last equation is satisfied only when m is a solution of the algebraic equation

$$m^2 - 6m + 9 = 0.$$

We get $m_1 = m_2 = 3$. Since $m_1 = m_2$ we obtain only one exponential solution

$$x_1(t) = e^{3t}.$$

The second solution is

$$x_2(t) = te^{3t}.$$

The general solution of the original equation is

$$x(t) = c_1x_1(t) + c_2x_2(t) = c_1e^{3t} + c_2te^{3t}$$

where c_1 and c_2 are constants.

Now we find c_1 and c_2 using the initial conditions

$$x(0) = c_1 = 6$$

$$x(0) = c_1 = -1$$

However, the constant cannot simultaneously be equal to different numbers, so the initial value problem has no solutions

Answer: this initial problem has no solutions.

Addition. If we assume that there is a misprint in the condition, that is, the condition is

$$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 9x = 0, \quad x(0) = 6, \quad x'(0) = -1$$

then we can find c_1 and c_2

$$x(0) = c_1 + 0 = 6 \quad \Rightarrow \quad c_1 = 6$$

$$x'(0) = 3c_1 + c_2 = -1 \quad \Rightarrow \quad c_2 = -19$$

Finally

$$x(t) = 6e^{3t} - 19te^{3t}$$

Answer: $x(t) = 6e^{3t} - 19te^{3t}$