Answer on Question #71893 – Math – Differential Equations

Question

dy/dx+ycotx=5e^cosx

Solution

The given equation

$$\frac{dy}{dx} + y\cot x = 5e^{\cos x}$$

is a linear differential equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x),$$

where $P(x) = \cot x,\ \ Q(x) = 5e^{\cos x}$.

To solve this equation we find the integrating factor

$$I(x) = e^{\int P(x)dx} = e^{\int \cot x dx}$$

Calculate the integral in the exponent:

$$\int \cot x dx = \int \frac{\cos x}{\sin x} dx = \int \frac{d(\sin x)}{\sin x} = \ln|\sin x|$$

So the integrating factor is

$$I(x) = e^{\ln|\sin x|} = \sin x$$

Multiply both sides of the given equation by $I(x) = \sin x$:

$$\sin x \frac{dy}{dx} + (\sin x \cdot \cot x)y = 5\sin x e^{\cos x}$$

$$\sin x \frac{dy}{dx} + (\cos x)y = 5\sin x e^{\cos x}$$

Using the Product Rule we get

$$\frac{d}{dx}(y\sin x) = 5\sin x e^{\cos x}$$

Integrate both sides of this equation :

$$\int \frac{d}{dx}(y\sin x)dx = \int 5\sin x e^{\cos x} dx,$$

$$y\sin x = 5\int e^{\cos x} d(-\cos x) = -5\int e^{\cos x} d(\cos x),$$

$$y\sin x = -5e^{\cos x} + c,$$

where c is an integration constant.

Dividing both sides by $\sin x$ we get the general solution of the original equation:

$$y = (-5e^{\cos x} + c)\csc x.$$

Answer: $y = (-5e^{\cos x} + c)\csc x.$

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