Answer on Question #71657 – Math – Differential Equations

$$(D^2 - 2DD' + D'^2)z = x^3$$

Solution

The auxiliary equation:

$$k^2 - 2k + 1 = 0$$

$$k_{1,2} = 1$$

Since the auxiliary equation has equal roots, then:

$$u = \varphi_1(y + kx) + x\varphi_2(y + kx) = \varphi_1(y + x) + x\varphi_2(y + x)$$

where φ_1, φ_2 are arbitrary functions.

Reference: *Differential Equations* by K. S. Rawat, page 331

Then particular integral:

$$p = \frac{1}{f(D, D')}\varphi(x, y)$$

Reference: *Differential Equations* by K. S. Rawat, page 334

So we have:

$$f(D, D') = D^2 - 2DD' + D'^2$$

$$\varphi(x, y) = x^3$$

$$p = \frac{1}{D^2 - 2DD' + D'^2}x^3 = \frac{1}{(D - D')^2}x^3 = \frac{1}{D^2\left(1 - \frac{D'}{D}\right)^2}x^3 = \frac{1}{D^2}\left(1 - \frac{D'}{D}\right)^{-2}x^3 =$$

$$= \frac{1}{D^2}\left(1 + \frac{2D'}{D} + \frac{3D'}{D^2} + \cdots\right)x^3 = \frac{1}{D^2}x^3 = \frac{x^5}{20}$$

The general solution of the given equation is $z = u + p$

Answer: $z = \varphi_1(y + x) + x\varphi_2(y + x) + \frac{x^5}{20}$