Answer on Question #71197 – Math – Calculus

Question

The demand function is $p = 85 - 0.05q$ and the average cost (in dollars per unit) is

$$\overline{c} = 2q^2 - 42q + \frac{210}{q}.$$

At what level should production be fixed in order to maximize profit? At what price does this occur, and what is the profit?

Solution

First let's find the cost function

$$c = q\overline{c} = 2q^3 - 42q^2 + 210$$

Find the revenue

$$R = pq = (85 - 0.05q)q = 85q - 0.05q^2$$

Find the profit

$$P = R - c$$
$$P = 85q - 0.05q^2 - 2q^3 + 42q^2 - 210$$
$$P = -2q^3 + 41.95q^2 + 85q - 210$$

Find the first derivative with respect to q

$$\frac{dP}{dq} = (-2q^3 + 41.95q^2 + 85q - 210)' = -6q^2 + 83.9q + 85$$

Find the critical point(s)

$$\frac{dP}{dq} = 0 => -6q^2 + 83.9q + 85 = 0$$

$$q = \frac{-83.9 \pm \sqrt{(83.9)^2 - 4(-6)(85)}}{2(-6)} = \frac{-83.9 \pm \sqrt{9079.21}}{-12}$$

$$q_1 = \frac{-83.9 - \sqrt{9079.21}}{-12} = \frac{83.9 + \sqrt{9079.21}}{12}$$

$$q_2 = \frac{-83.9 + \sqrt{9079.21}}{-12} = \frac{83.9 - \sqrt{9079.21}}{12}$$

$$If\ q < \frac{83.9 - \sqrt{9079.21}}{12}, \frac{dP}{dq} < 0, P\ decreases$$

$$If\ \frac{83.9 - \sqrt{9079.21}}{12} < q < \frac{83.9 + \sqrt{9079.21}}{12}, \frac{dP}{dq} > 0, P\ increases$$

$$If\ q > \frac{83.9 + \sqrt{9079.21}}{12}, \frac{dP}{dq} < 0, P\ decreases$$

The profit function has a local maximum at $q = \frac{83.9 + \sqrt{9079.21}}{12} \approx 14.932.$

$P(0) = -210$

$P(14) = -2(14)^3 + 41.95(14)^2 + 85(14) - 210 = 3714.2$

$P(15) = -2(15)^3 + 41.95(15)^2 + 85(15) - 210 = 3753.75$

Then $p = 85 - 0.05(15) = 84.25$.
Answer: Profit will be maximized when there are 15 units made, at the price of \$84.25 per unit, and the maximum profit is $\$3753.75$.