Answer on Question #71176 – Math – Calculus

Question

Trace the curve $xy^2 = a^2(a-x), a > 0$

Solution

1. If $x \le 0$: $xy^2 \le 0, a^2(a-x) > 0$

$xy^2 \ne a^2(a-x)$

$x \in (0, a]$

$y \in (-\infty, \infty)$

2. Symmetry

2.1. Symmetry about x −axis

$$x(-y)^2 = xy^2 = a^2(a-x)$$

Curve is symmetric about x −axis.

2.2. Symmetry about y −axis

$$-xy^2 \ne a^2(a+x)$$

Curve is not symmetric about y −axis.

2.3. Symmetry about line $y = x$

Interchange x with y and y with x

$$xy^2 = a^2(a-x) \to yx^2 = a^2(a-y)$$

Curve is not symmetric about line $y = x$.

2.4. Symmetry in opposite quadrants

Curve is not symmetric in opposite quadrants.

3. Origin and tangents

The curve is not passing through the origin

4. Intersection with coordinate axes

Since $x \in (0, a]$, the curve has no y −intercept.

$x - intercept: y = 0 => \ 0 = a^2(a-x), x = a$

$Point\ (a, 0)$

5. Asymptotes

$$\lim_{x\to 0^+} y^2 = \lim_{x\to 0^+} \frac{a^2(a-x)}{x} = \lim_{x\to 0^+} \left(a^2\left(\frac{a}{x} - 1\right)\right) = +\infty$$

Vertical asymptote: $x = 0$.

There is no horizontal asymptote.

There is no slant asymptote.

6. Find the first derivative with respect to x

$(xy^2)' = (a^2(a-x))'$

$y^2 + 2xyy' = -a^2$

$y' = -\frac{y^2 + a^2}{2xy}$

$x > 0$

y' is not defined, when $y = 0$

If $y > 0, y' < 0$

If $y < 0, y' > 0$

The tangent is vertical at $Point\ (a, 0)$

Find the second derivative

$$y'' = \left(-\frac{y^2 + a^2}{2xy}\right)' = -\frac{2yy'xy - (y + xy')(y^2 + a^2)}{2x^2y^2} =$$

$$= -\frac{-y(y^2 + a^2) - (y - \frac{y^2 + a^2}{2y})(y^2 + a^2)}{2x^2y^2} =$$

$$= (y^2 + a^2)\frac{y + y - \frac{y^2 + a^2}{2y}}{2x^2y^2} = (y^2 + a^2)\frac{3y^2 - a^2}{4x^2y^3}$$

y'' is not defined, when $y = 0$

$$y'' = 0 \Rightarrow (y^2 + a^2)\frac{3y^2 - a^2}{4x^2y^3} = 0$$

$y^2 = \frac{1}{3}a^2$

$x\left(\frac{1}{3}a^2\right) = a^2(a - x)$

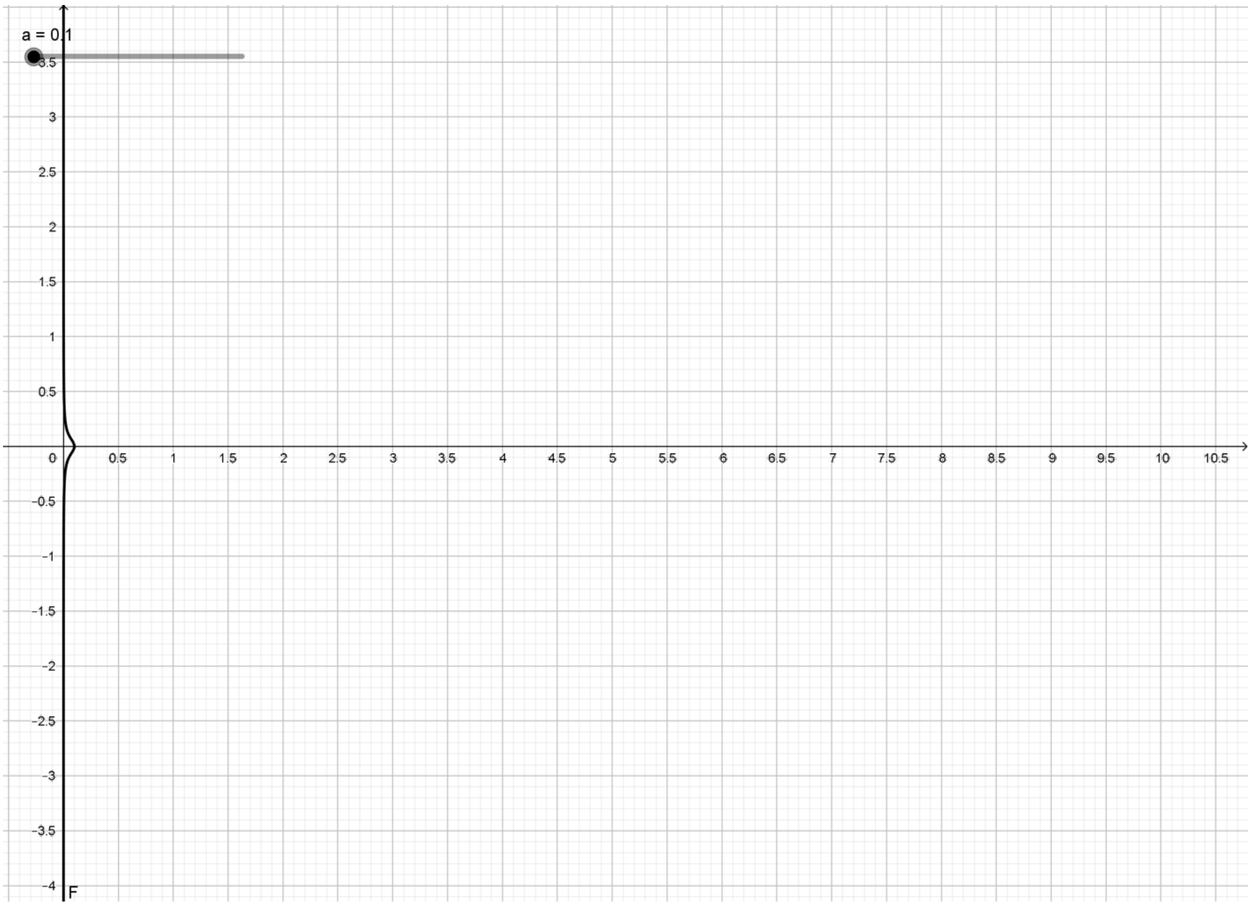
$x = 3a - 3x$

$x = \frac{3}{4}a$

$If\ 0 < y < \frac{a}{\sqrt{3}}, y'' < 0, the\ graph\ concave\ down$

$If\ y > \frac{a}{\sqrt{3}}, y'' > 0, the\ graph\ concave\ up$

$Inflection\ points: \left(\frac{3}{4}a, -\frac{a}{\sqrt{3}}\right), \left(\frac{3}{4}a, \frac{a}{\sqrt{3}}\right)$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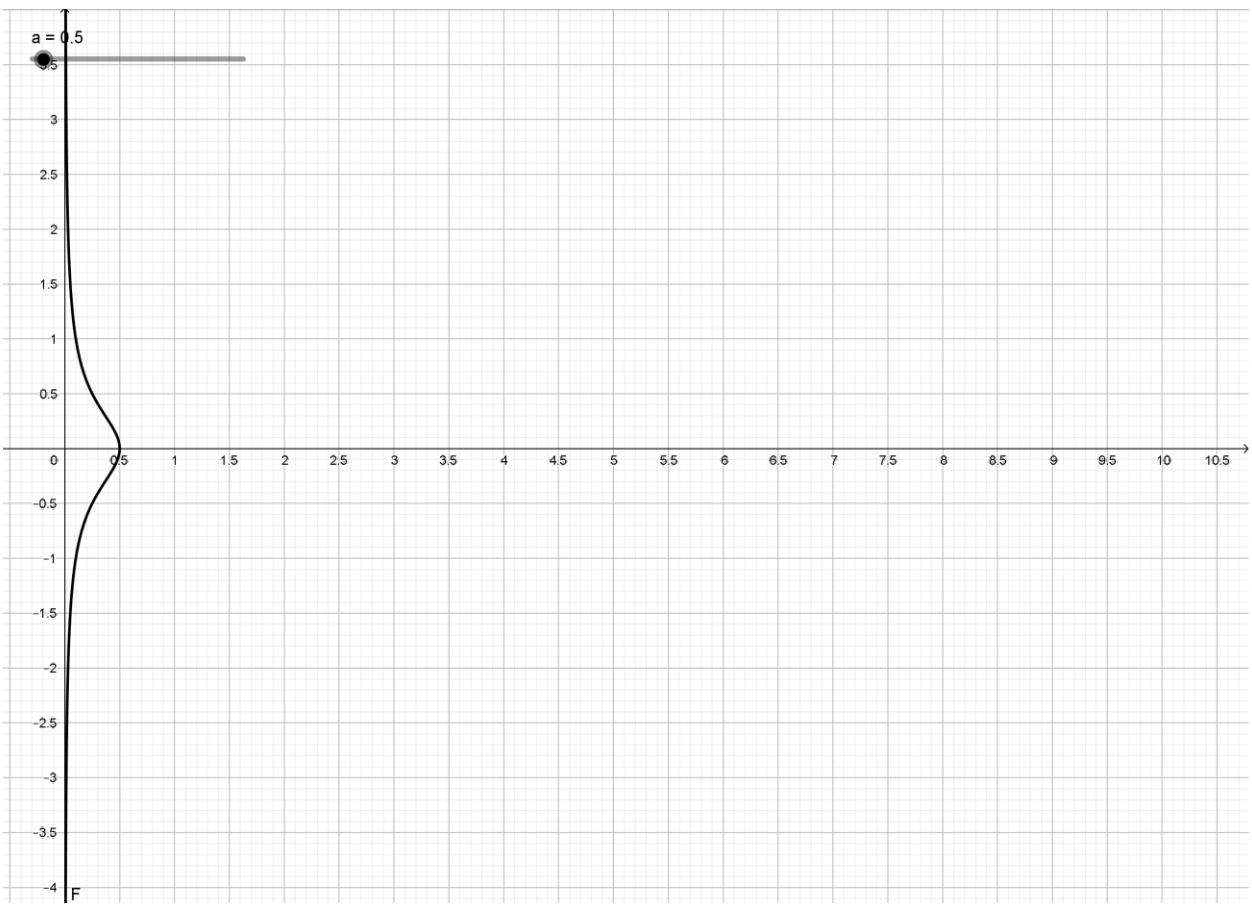
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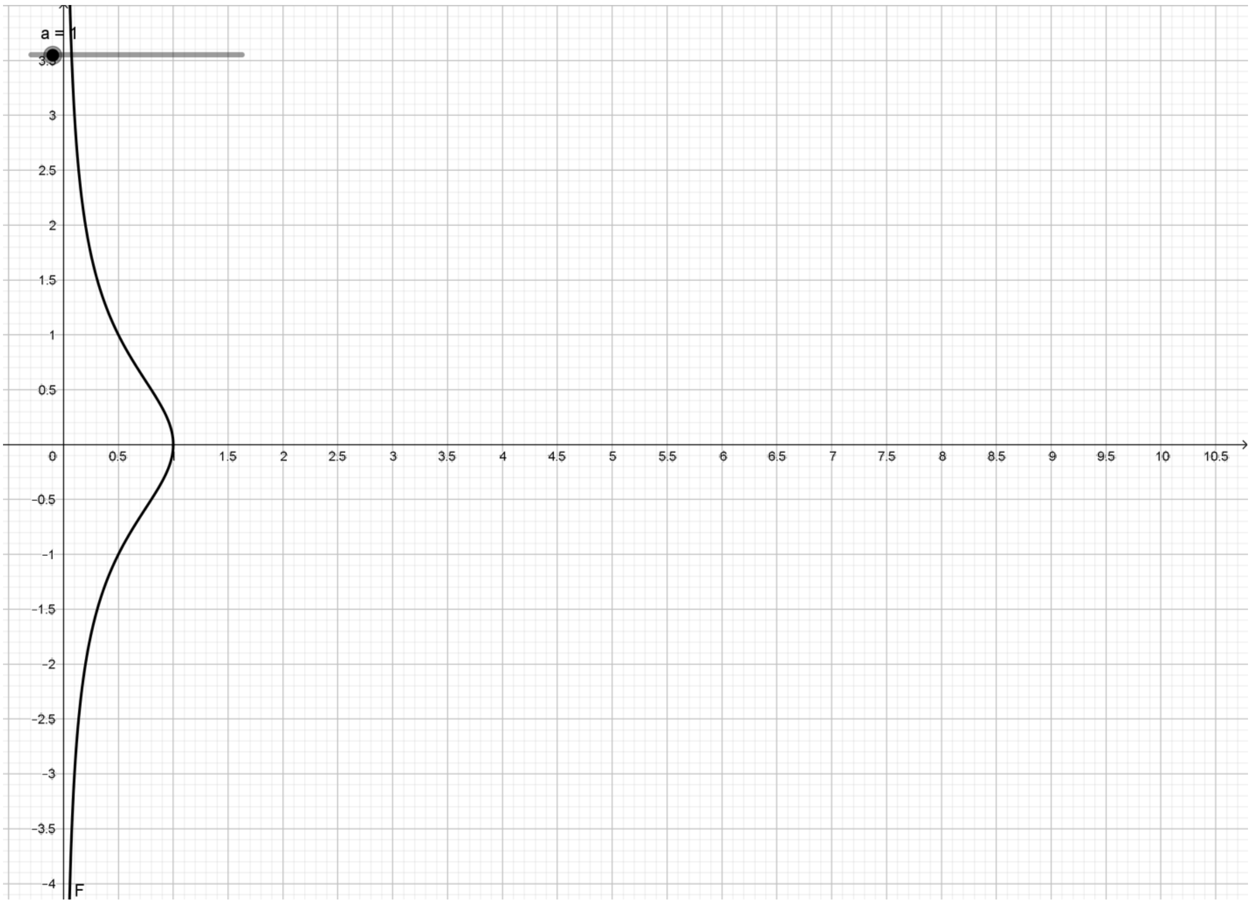

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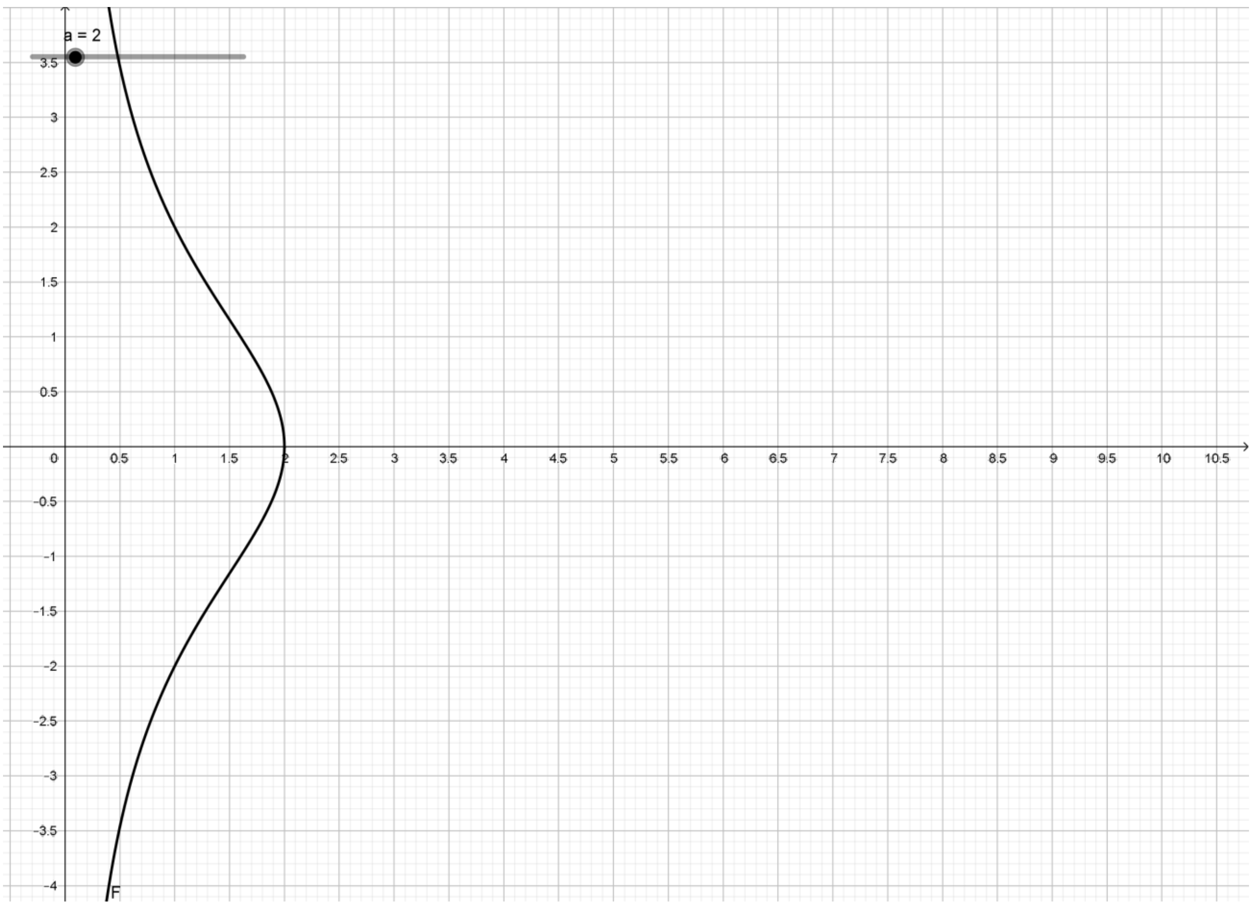

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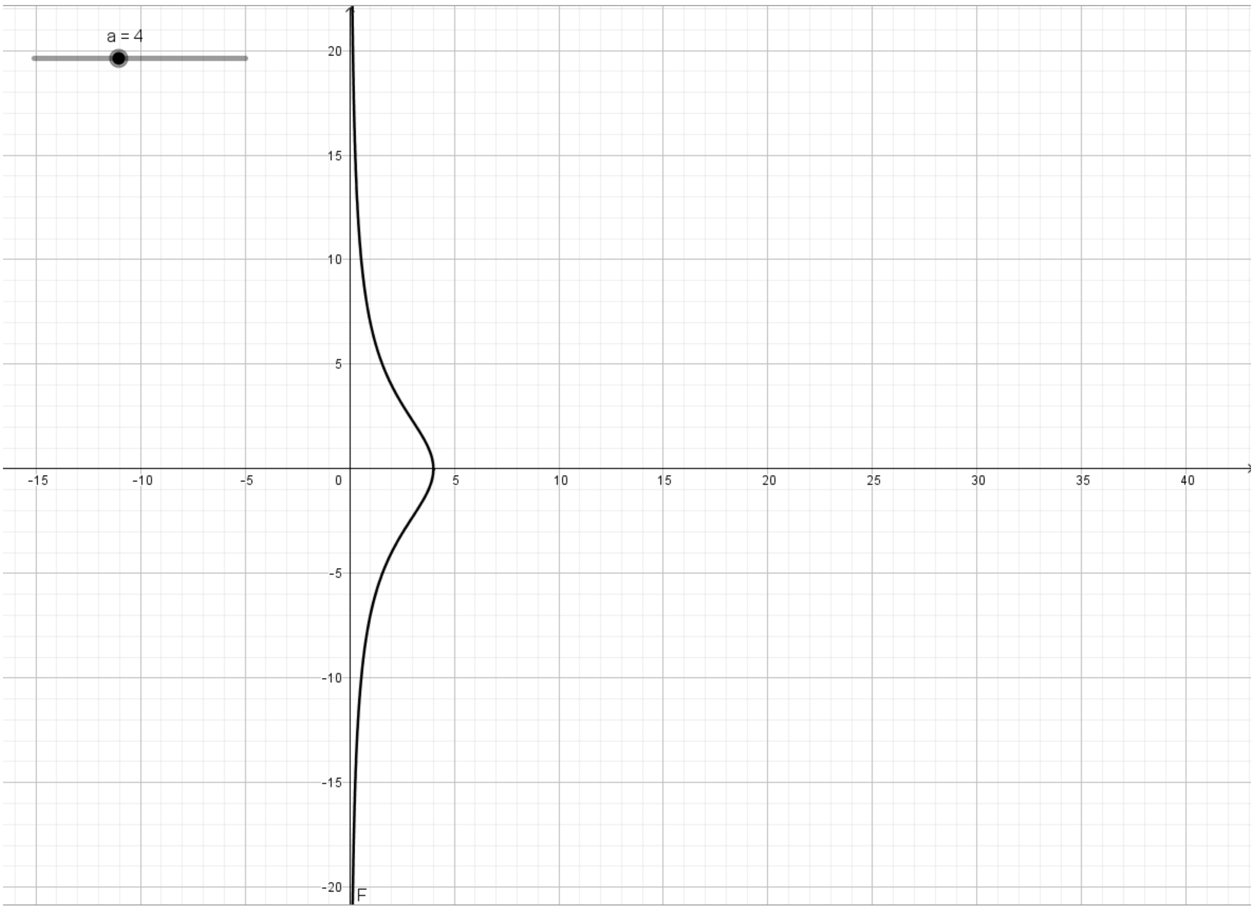
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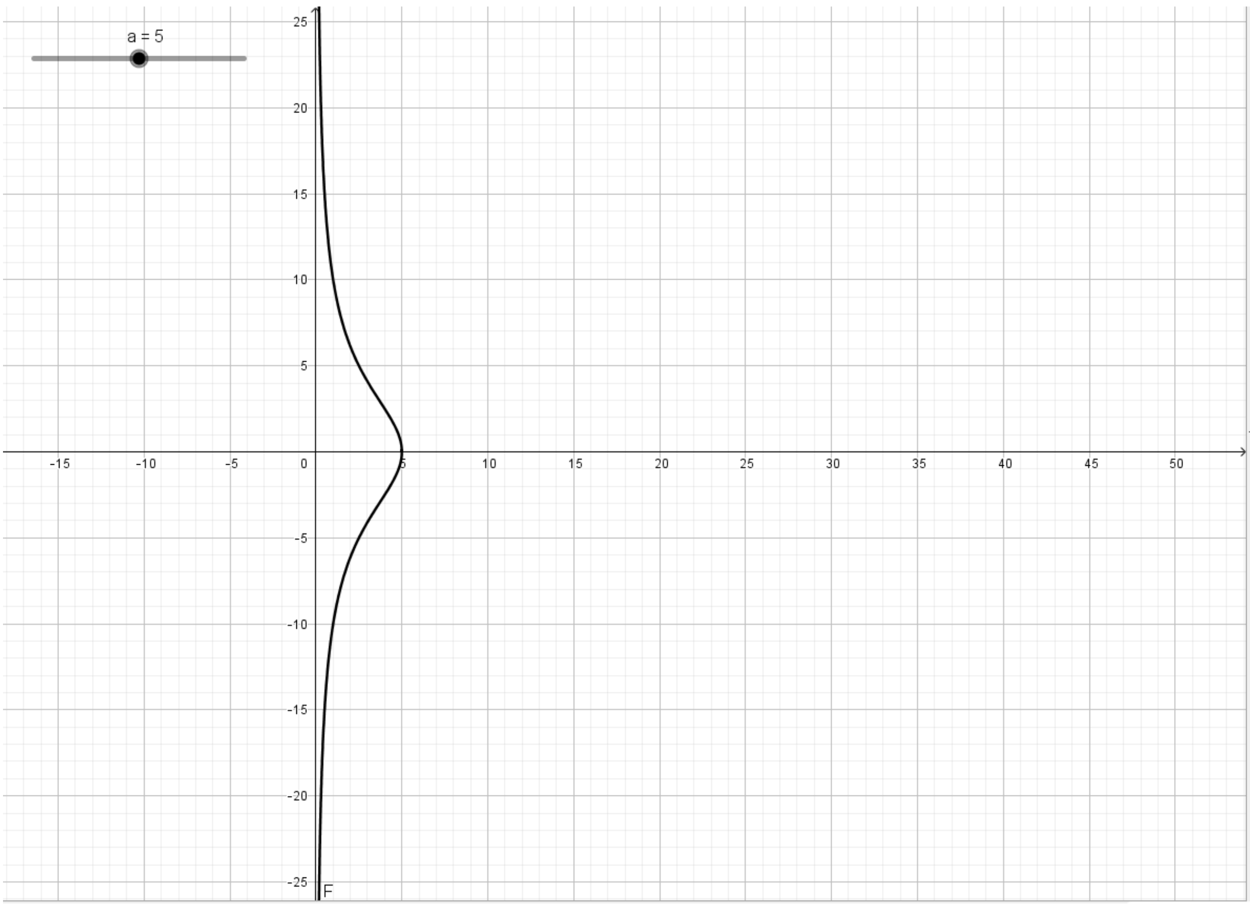
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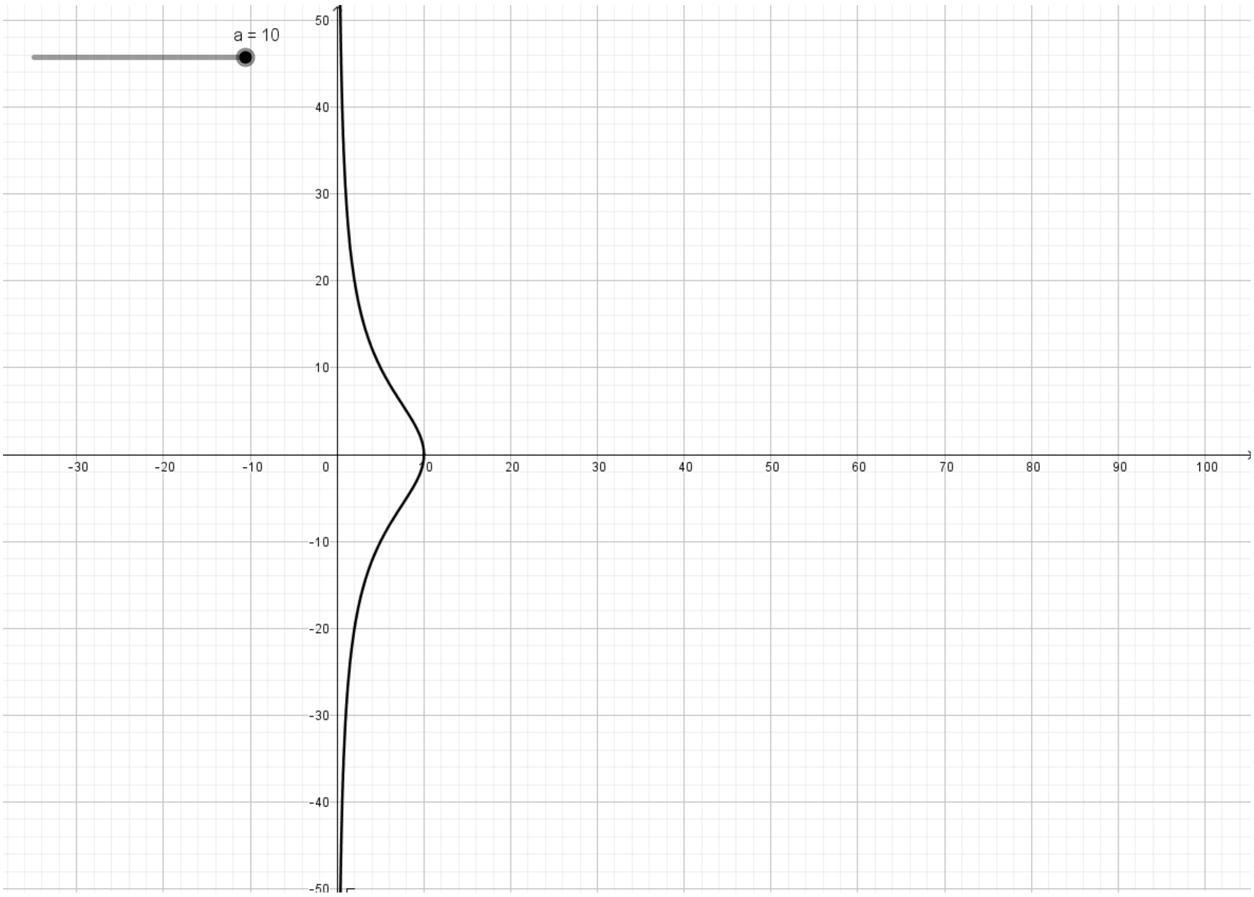
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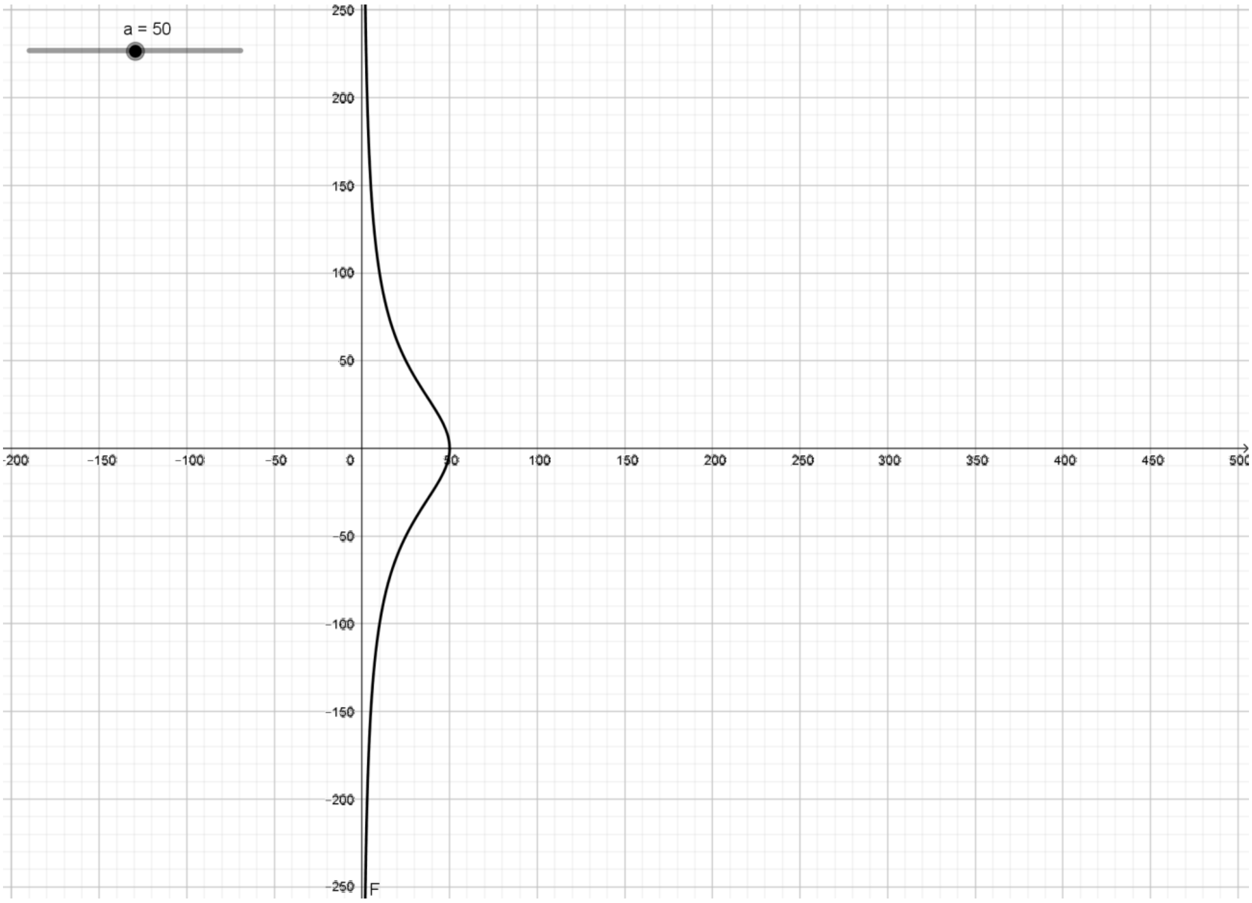
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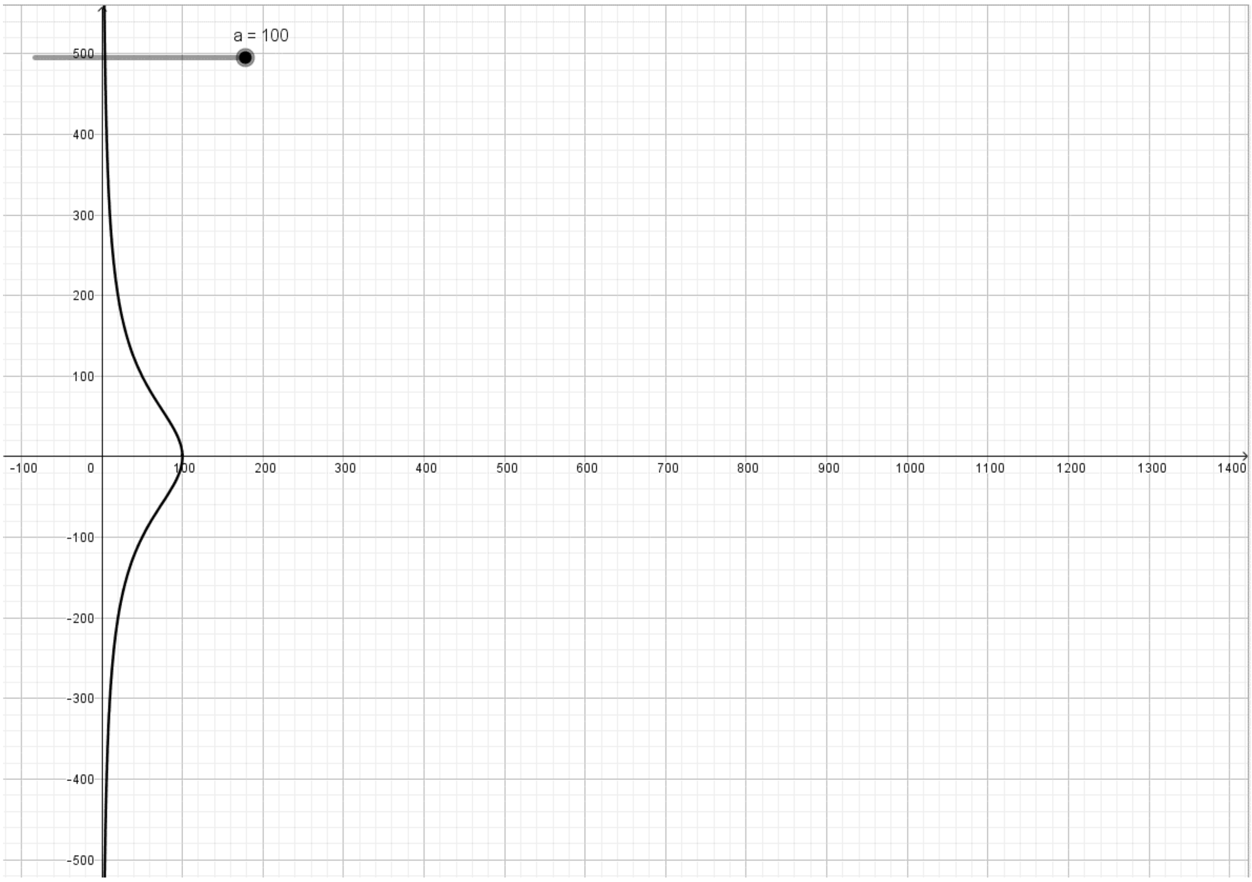

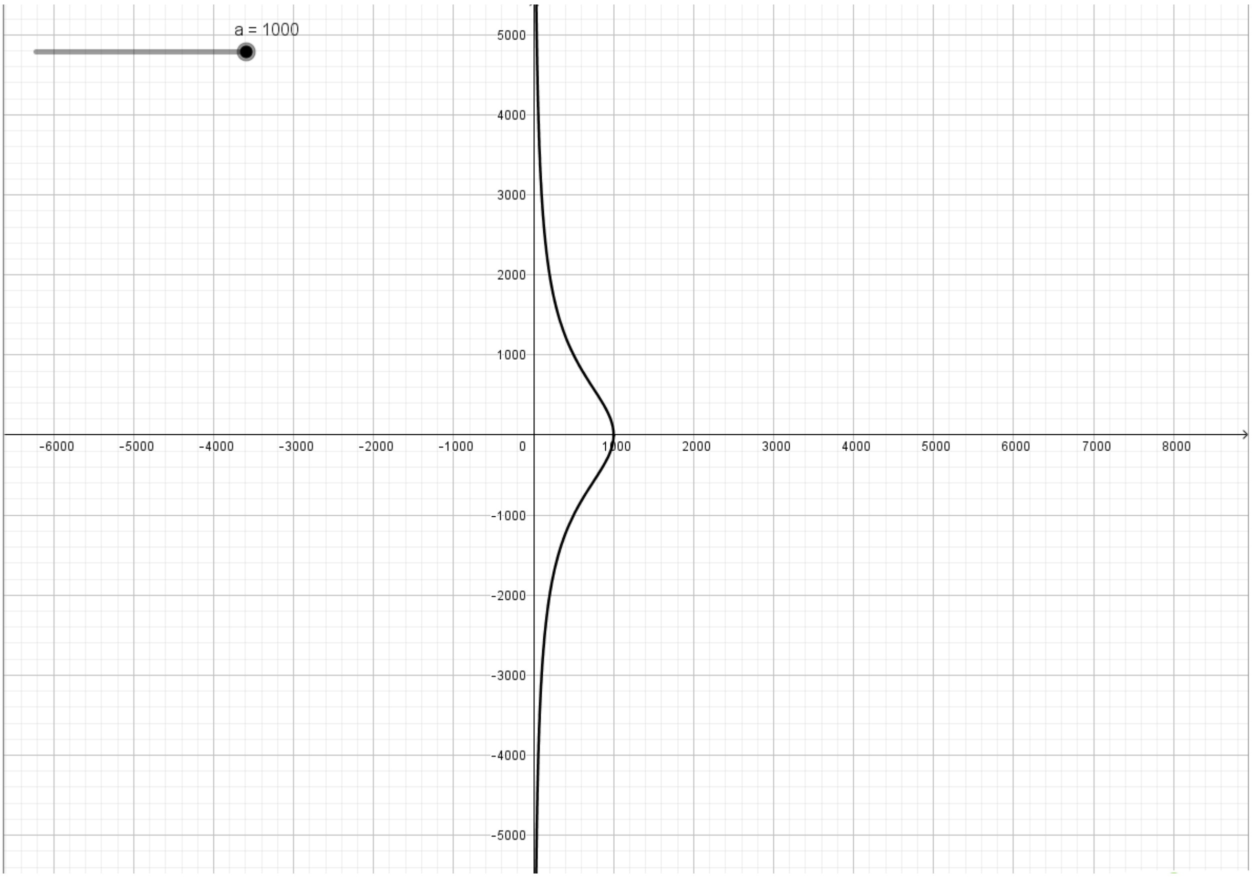

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