Question

Find the radius of the sphere which passes through the points $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$.

Solution

The general equation of a sphere is
$$x^2 + y^2 + z^2 + 2gx + 2fy + 2hz + c = 0$$
Since it passes through the point $(0,0,0)$
$$0 + 0 + 0 + 0 + 0 + 0 + c = 0 => c = 0$$
$$x^2 + y^2 + z^2 + 2gx + 2fy + 2hz = 0$$
Since it passes through the point $(1,0,0)$
$$1 + 0 + 0 + 2g(1) + 0 + 0 = 0 => g = -\frac{1}{2}$$
$$x^2 + y^2 + z^2 - x + 2fy + 2hz = 0$$
Since it passes through the point $(0,1,0)$
$$0 + 1 + 0 - 0 + 2f(1) + 0 = 0 => f = -\frac{1}{2}$$
$$x^2 + y^2 + z^2 - x - y + 2hz = 0$$
Since it passes through the point $(0,0,1)$
$$0 + 0 + 1 - 0 - 0 + 2h(1) = 0 => h = -\frac{1}{2}$$
$$x^2 + y^2 + z^2 - x - y - z = 0$$
We have that
$$x^2 - 2\left(\frac{1}{2}\right)x + \frac{1}{4} + y^2 - 2\left(\frac{1}{2}\right)y + \frac{1}{4} + z^2 - 2\left(\frac{1}{2}\right)x + \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$
$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 + \left(z - \frac{1}{2}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$$

Then
$$R = \frac{\sqrt{3}}{2}$$

Answer: the radius of the sphere equals $\frac{\sqrt{3}}{2}$.