

Prove or disprove that:

- $A - B \subseteq A - C - B$
- $(A - B) \cup (B - C) \cup (C - A) = (A \cup B \cup C) - (A \cap B \cap C)$

**Solution:**

- Answer:** That is wrong.

Let  $A$  be  $\{1,2,3\}$ ,  $B=\{1\}$ ,  $C=\{2\}$ .  $A-B=\{2,3\}$ ,  $A-C-B=\{3\}$ .  $\{2,3\} \not\subseteq \{3\}$ .

- Answer:** That is true.

Let's denote  $A_1 = A - B - C$ ,  $B_1 = B - A - C$ ,  $C_1 = C - A - B$ ,  $AB = A \cap B - C$ ,  $AC = A \cap C - B$ ,  $BC = B \cap C - A$ ,  $ABC = A \cap B \cap C$ .

$A - B = A_1 \cup AC$ ,  $B - C = B_1 \cup AB$ ,  $A - C = A_1 \cup BC \implies (A - B) \cup (B - C) \cup (C - A) = A_1 \cup AC \cup B_1 \cup AB \cup A_1 \cup BC$ .

$(A \cup B \cup C) - (A \cap B \cap C) = A_1 \cup B_1 \cup C_1 \cup AB \cup BC \cup AC$ .

Thus  $(A - B) \cup (B - C) \cup (C - A) = (A \cup B \cup C) - (A \cap B \cap C)$

