Answer on Question #71152 – Math – Calculus

Question

Experiments in an electrical engineering laboratory appear to be related by the following hyperbolic and trognometric relationship.

$$3\sinh x + 2\cosh x = \tan(81.87°)$$

a) Using your knowledge of exponential representation of hyperbolic function, find all values of a which satisfy the equation.

b) Verify your solutions using the standard hyperbolic and trigonometric functions.

Solution

a)

$$3\sinh x + 2\cosh x = \tan(81.87°);$$

$$\tan(81.87°) = 7$$

$$3\sinh x + 2\cosh x = 7$$

$$\sinh x = \frac{e^x - e^{-x}}{2}; \cosh x = \frac{e^x + e^{-x}}{2};$$

$$\frac{3}{2}(e^x - e^{-x}) + (e^x + e^{-x}) = 7;$$

$$\frac{5}{2}e^x - \frac{1}{2}e^{-x} = 7;$$

Multiplying through by $2e^x$

$$5e^{2x} - 14e^x - 1 = 0;$$

Substituting

$$e^x = t, \quad t > 0.$$

Then

$$5t^2 - 14t - 1 = 0;$$

$$D = 14^2 + 4 \cdot 1 \cdot 5 = 216 = 36 \cdot 6;$$

$$t_1 = \frac{7+3\sqrt{6}}{5};\ t_2 = \frac{7-3\sqrt{6}}{5} < 0 - does\ not\ satisfy\ the\ condition\ t > 0;$$

$$e^x = \frac{7+3\sqrt{6}}{5};$$

$$x = \ln\frac{7+3\sqrt{6}}{5} \approx 1.0542.$$

b)

Verification:

$$\sinh(1.0542) = 1.26; \quad \cosh(1.0542) = 1.609;$$

$$3 \cdot 1.26 + 2 \cdot 1.609 = 6.998 \approx 7.$$