Answer on Question #71149 – Math – Functional Analysis

Question

Which of the following sets are countable or uncountable.
(i) set of negative integers
(ii) set of rational numbers

Solution

(i) We can list the integers in a sequence:

$$-1, -2, -3, -4, \ldots$$

Therefore, the sequence can be numbered, $-1 \to 1$, $-2 \to 2$, $-3 \to 3$, $\ldots, -n \to n$. We have obtained a bijective function $f(n) = -n$. Hence the set of negative integers is countable.

(ii) To show that rational numbers are countable it is necessary to find a one-to-one correspondence between the elements of the set and the set of natural numbers. By definition, a rational number $\frac{p}{q}$, where p, q integer and $q \neq 0$. We write the rational numbers in the form of a sequence, writing down all possible combinations for p and q not exceeding in absolute value 1, then 2 and so on (except for the repetition of numbers and writing first the numbers with $p > q$ then $q > p$). As a result, we obtain the sequence

$$\frac{0}{1}, \frac{1}{1}, \frac{-1}{1}, \frac{1}{2}, \frac{-1}{2}, \frac{2}{1}, \frac{-2}{1}, \frac{1}{3}, \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}, \frac{3}{1}, \frac{3}{2}, \frac{-3}{1}, \frac{-3}{2}, \frac{1}{4}, \frac{3}{4}, \frac{-1}{4}, \frac{-3}{4}, \frac{4}{1}, \frac{4}{3}, \frac{-4}{1}, \frac{-4}{3}, \ldots$$

The sequence can be numbered, hence it is one-to-one correspondence between the elements of the set and the set of natural numbers. Hence set of rational numbers is countable.

Answer: **(i)** countable, **(ii)** countable.