Answer on Question #71109 – Math – Statistics and Probability

Question

The average number of patients admitted per day to the emergency room of a small hospital is 2.5. If, on any given day, there is/are only 9 bed(s) available for new patients, what is the probability that the hospital will not have enough beds to accommodate its newly admitted patients?

Solution

It is given that the average (mean) number of patients admitted per day to the emergency room of a small hospital is 2.5.
Let $X =$ the number of patients admitted to the emergency room on a given day.
We have that X follows a Poisson distribution with mean $\lambda = 2.5$ patients per day.
Define the probability mass function of X.

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2, \ldots$$

$$P(X = x) = \frac{e^{-2.5}2.5^x}{x!}, x = 0, 1, 2, \ldots$$

Since there are only 9 beds available for new patients, the deficiency of beds will occur when the number of patients is more than 9.
We are looking for $P(x > 9) = 1 - P(X \leq 9)$

$$P(X = 0) = \frac{e^{-2.5}2.5^0}{0!} = e^{-2.5} \approx 0.082085$$

$$P(X = 1) = \frac{e^{-2.5}2.5^1}{1!} \approx 0.205212$$

$$P(X = 2) = \frac{e^{-2.5}2.5^2}{2!} \approx 0.256516$$

$$P(X = 3) = \frac{e^{-2.5}2.5^3}{3!} \approx 0.213763$$

$$P(X = 4) = \frac{e^{-2.5}2.5^4}{4!} \approx 0.133602$$

$$P(X = 5) = \frac{e^{-2.5}2.5^5}{5!} \approx 0.066801$$

$$P(X = 6) = \frac{e^{-2.5}2.5^6}{6!} \approx 0.027834$$

$$P(X = 7) = \frac{e^{-2.5}2.5^7}{7!} \approx 0.009941$$

$$P(X = 8) = \frac{e^{-2.5}2.5^8}{8!} \approx 0.003106$$

$$P(X = 9) = \frac{e^{-2.5}2.5^9}{9!} \approx 0.000863$$

$$P(X > 9) = 1 - P(X \leq 9) = 1 - P(X = 0) - P(X = 1) - P(X = 2) -$$

$-P(X=3)-P(X=4)-P(X=5)-P(X=6)-P(X=7)-P(X=8)-$
$-P(X=9)$

$P(X>9)\approx 1-0.082085-0.205212-0.256516-0.213763-0.133602-$
$-0.066801-0.027834-0.009941-0.003106-0.000863=0.000277.$
Answer: 0.000277.