## Answer on Question \#71100 - Math - Analytic Geometry

## Question

A small television has a picture with a diagonal measure of 10 inches and a viewing area of 48 square inches. Find the width and length of the screen.

## Solution

The screen is a rectangle. Let $x$ and $y$ be sides of the rectangle (the width and the length of the screen), then the diagonal of the screen is a diagonal of the rectangle, its measure is $d=10$.

By Pythagorean Theorem,

$$
d^{2}=x^{2}+y^{2}
$$

The viewing area $S=48$ is the area of rectangle:

$$
S=x y .
$$

Solve system of equations:

$$
\begin{aligned}
& \left\{\begin{array} { l } 
{ x ^ { 2 } + y ^ { 2 } = 1 0 ^ { 2 } } \\
{ x y = 4 8 }
\end{array} \Leftrightarrow \left\{\begin{array} { l } 
{ x ^ { 2 } + y ^ { 2 } = 1 0 0 , } \\
{ x y = 4 8 }
\end{array} \Leftrightarrow \left\{\begin{array}{l}
x^{2}-2 x y+y^{2}=100-2 \cdot 48, \\
x y=48
\end{array} \Leftrightarrow\right.\right.\right. \\
& \Leftrightarrow\left\{\begin{array}{l}
(x-y)^{2}=4, \\
x y=48
\end{array}, \Leftrightarrow\left\{\begin{array} { l } 
{ x - y = \pm 2 , } \\
{ x y = 4 8 }
\end{array} \Leftrightarrow \left\{\begin{array} { l } 
{ x = y \pm 2 , } \\
{ x y = 4 8 }
\end{array} \Leftrightarrow \left[\begin{array}{l}
\left\{\begin{array}{l}
x=y+2, \\
(y+2) y=48
\end{array}\right. \\
\left\{\begin{array}{l}
x=y-2, \\
(y-2) y=48
\end{array}\right.
\end{array} \Leftrightarrow\right.\right.\right.\right. \\
& \Leftrightarrow\left[\begin{array}{l}
\left\{\begin{array}{l}
x=y+2, \\
y^{2}+2 y-48=0 \\
x=y-2, \\
y^{2}-2 y-48=0
\end{array}\right.
\end{array}\right. \\
& \begin{array}{l|l}
y^{2}+2 y-48=0 \\
D=b^{2}-4 a c=2^{2}-4 \cdot 1 \cdot(-48)=196 & \begin{array}{l}
y^{2}-2 y-48=0 \\
D=b^{2}-4 a c=(-2)^{2}-4 \cdot 1
\end{array} \cdot(-48)=196 \\
y=\frac{-b \pm \sqrt{D}}{2 a}=\frac{-2 \pm 14}{2}=\left[\begin{array}{c}
6, \\
-8
\end{array}\right. & y=\frac{-b \pm \sqrt{D}}{2 a}=\frac{2 \pm 14}{2}=\left[\begin{array}{c}
8, \\
-6
\end{array}\right.
\end{array}
\end{aligned}
$$

We won't consider $y=-8$ and $y=-6$ because $y$ is a side of rectangle and must be positive.

$$
\left[\begin{array} { l } 
{ \{ \begin{array} { l } 
{ x = y + 2 , } \\
{ y ^ { 2 } + 2 y - 4 8 = 0 } \\
{ x = y - 2 , } \\
{ y ^ { 2 } - 2 y - 4 8 = 0 }
\end{array} }
\end{array} \Leftrightarrow \left[\begin{array} { l } 
{ \{ \begin{array} { l } 
{ x = y + 2 , } \\
{ y = 6 }
\end{array} } \\
{ \{ \begin{array} { l } 
{ x = y - 2 , } \\
{ y = 8 }
\end{array} }
\end{array} \Leftrightarrow \left[\begin{array}{l}
\left\{\begin{array}{l}
x=8 \\
y=6
\end{array}\right. \\
\left\{\begin{array}{l}
x=6 \\
y=8
\end{array}\right.
\end{array}\right.\right.\right.
$$

The system has two solutions: $x_{1}=8, y_{1}=6$ and $x_{2}=6, y_{2}=8$. It means that sides of required rectangle are 6 and 8 .

Thus, the width and length of the screen are 6 and 8 inches.

Answer: 6 and 8 inches.

