Answer on Question #71100 – Math – Analytic Geometry

Question

A small television has a picture with a diagonal measure of 10 inches and a viewing area of 48 square inches. Find the width and length of the screen.

Solution

The screen is a rectangle. Let x and y be sides of the rectangle (the width and the length of the screen), then the diagonal of the screen is a diagonal of the rectangle, its measure is d = 10.

By Pythagorean Theorem,

$$d^2 = x^2 + y^2.$$

The viewing area S = 48 is the area of rectangle:

$$S = xy$$
.

Solve system of equations:

$$\begin{cases} x^{2} + y^{2} = 10^{2}, \\ xy = 48 \end{cases} \Leftrightarrow \begin{cases} x^{2} + y^{2} = 100, \\ xy = 48 \end{cases} \Leftrightarrow \begin{cases} x^{2} - 2xy + y^{2} = 100 - 2 \cdot 48, \\ xy = 48 \end{cases} \Leftrightarrow \\ xy = 48 \end{cases} \Leftrightarrow \\ \begin{cases} x = y + 2, \\ (y + 2)y = 48 \\ xy = 48 \end{cases} \Leftrightarrow \\ \begin{cases} x = y + 2, \\ (y - 2)y = 48 \\ xy = 48 \end{cases} \Leftrightarrow \\ \begin{cases} x = y - 2, \\ (y - 2)y = 48 \\ xy = 48 \end{cases} \Leftrightarrow \\ \begin{cases} x = y - 2, \\ (y^{2} + 2y - 48 = 0) \\ x^{2} + 2y - 48 = 0 \\ y^{2} + 2y - 48 = 0 \\ y^{2} - 2y - 48 = 0 \end{cases} \Rightarrow \\ y^{2} - 2y - 48 = 0 \\ y = \frac{-b \pm \sqrt{D}}{2a} = \frac{-2 \pm 14}{2} = \begin{bmatrix} 6, \\ -8 \end{cases} \Rightarrow \\ y = \frac{-b \pm \sqrt{D}}{2a} = \frac{-2 \pm 14}{2} = \begin{bmatrix} 6, \\ -8 \end{cases} \Rightarrow \\ y = \frac{-b \pm \sqrt{D}}{2a} = \frac{2 \pm 14}{2} = \begin{bmatrix} 8, \\ -6 \end{bmatrix} \end{cases}$$

We won't consider y = -8 and y = -6 because y is a side of rectangle and must be positive.

$$\begin{cases} x = y + 2, \\ y^2 + 2y - 48 = 0 \\ x = y - 2, \\ y^2 - 2y - 48 = 0 \end{cases} \Leftrightarrow \begin{cases} x = y + 2, \\ y = 6 \\ x = y - 2, \\ y = 8 \end{cases} \Leftrightarrow \begin{cases} x = 8, \\ y = 6 \\ x = 6, \\ y = 8 \end{cases}$$

The system has two solutions: $x_1 = 8$, $y_1 = 6$ and $x_2 = 6$, $y_2 = 8$. It means that sides of required rectangle are 6 and 8.

Thus, the width and length of the screen are 6 and 8 inches.

Answer: 6 and 8 inches.

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