Answer on Question #71039 – Math – Calculus

Question

Integrate $\int x(\ln x)^2\,dx$

Solution

Using integration by parts:

$$\int udv = uv - \int vdu$$

Let

$u = (\ln x)^2$, $dv = xdx$, then

$$du = \frac{2\ln x}{x}dx, v = \int xdx = \frac{x^2}{2}$$

$$\int x(\ln x)^2\,dx = \frac{x^2}{2}(\ln x)^2 - \int \frac{x^2}{2}\cdot\frac{2\ln x}{x}dx = \frac{x^2}{2}(\ln x)^2 - \int x\ln x dx$$

Using integration by parts again:

Let

$u = \ln x$, $dv = xdx$, then

$$du = \frac{1}{x}dx, v = \int xdx = \frac{x^2}{2}$$

$$\int x\ln x dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}dx = \frac{x^2}{2}\ln x - \int \frac{1}{2}dx = \frac{x^2}{2}\ln x - \frac{x^2}{4} + c$$

So

$$\int x(\ln x)^2\,dx = \frac{x^2}{2}(\ln x)^2 - \left(\frac{x^2}{2}\ln x - \frac{x^2}{4} + c\right) = \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{x^2}{4} + c$$

Answer: $\int x(\ln x)^2\,dx = \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{x^2}{4} + c$