Answer on Question #71038 – Math – Differential Equations

Question

$(1+yx)xdy+(1-yx)ydx=0$

Solution

$P(x,y)=(1-yx)y,\ \ \frac{\partial P}{\partial y}=1-yx-xy=1-2xy$

$Q(x,y)=(1+yx)x,\ \ \frac{\partial Q}{\partial x}=1+yx+yx=1+2xy$

$\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}=1-2xy-(1+2xy)=-4xy$

$Qy-Px=(1+xy)xy-(1-xy)xy=2(xy)^2$

$\frac{1}{Qy-Px}\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)=\frac{-4xy}{2(xy)^2}=-\frac{2}{xy}=f(xy)$

Integrating factor: $M(x,y)=M(xy)$

$$M\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)=Q\frac{\partial M}{\partial x}-P\frac{\partial M}{\partial y}$$

$\frac{\partial M}{\partial x}=\frac{dM}{d(xy)}y,\ \ \frac{\partial M}{\partial y}=\frac{dM}{d(xy)}x$

$$M(-4xy)=(1+yx)xy\frac{dM}{d(xy)}-(1-yx)yx\frac{dM}{d(xy)}$$

$$M(-4xy)=2(xy)^2\frac{dM}{d(xy)}$$

$$\frac{dM}{M}=-2\frac{d(xy)}{xy}$$

$$\int\frac{dM}{M}=\int-2\frac{d(xy)}{xy}$$

$$M=\frac{1}{(xy)^2}$$

We have the total differential

$\frac{1}{(xy)^2}(1+yx)xdy+\frac{1}{(xy)^2}(1-yx)ydx=0$

$\left(\frac{1}{xy^2}+\frac{1}{y}\right)dy+\left(\frac{1}{x^2y}-\frac{1}{x}\right)dx=0$

$P_1(x,y)=\frac{1}{x^2y}-\frac{1}{x},\ \ \frac{\partial P_1}{\partial y}=-\frac{1}{x^2y^2}$

$Q_1(x,y)=\frac{1}{xy^2}+\frac{1}{y},\ \ \frac{\partial Q_1}{\partial x}=-\frac{1}{x^2y^2}$

$$\frac{\partial P_1}{\partial y} = \frac{\partial Q_1}{\partial x}$$

$$dF(x, y) = \left(\frac{1}{x^2 y} - \frac{1}{x}\right) dx + \left(\frac{1}{xy^2} + \frac{1}{y}\right) dy$$

$$F = \int \left(\frac{1}{x^2 y} - \frac{1}{x}\right) dx = -\frac{1}{xy} - \ln|x| + \varphi(y)$$

$$\frac{\partial F}{\partial y} = \frac{1}{xy^2} + \varphi'_y(y) = \frac{1}{xy^2} + \frac{1}{y}$$

$$\varphi(y) = \int \left(\frac{1}{y}\right) dy = \ln|y| - C$$

Thus,

$$F(x, y) = -\frac{1}{xy} - \ln|x| + \ln|y| - C$$

$$F(x, y) = -\frac{1}{xy} + \ln\left|\frac{y}{x}\right| - C$$

or

$$-\frac{1}{xy} + \ln\left|\frac{y}{x}\right| - C = 0$$

Answer: $-\frac{1}{xy} + \ln\left|\frac{y}{x}\right| - C = 0.$