Answer on Question #71028 – Math – Calculus

Question

Using integration by parts to solve

1) $\int x\sin 2x dx$

2) $\int x \text{ arctan} x dx$

3) $\int \ln x/(1+x)^2 dx$

4) $\int (\ln x)^2 dx$

Solution

The formula for integration by parts is

$\int u dv = uv - \int v du$

1) $\int x\sin 2x dx$

Let

$u = x, \qquad dv = \sin 2x\, dx = d\left(-\frac{1}{2}\cos 2x\right)$

then

$du = dx, \qquad v = -\frac{1}{2}\cos 2x$

Integrating by parts, we get

$\int x\sin 2x dx = x\left(-\frac{1}{2}\cos 2x\right) - \int\left(-\frac{1}{2}\cos 2x\right)dx = -\frac{1}{2}x\cos 2x + \frac{1}{2}\int \cos 2x dx$

$= -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C$

2) $\int x \text{ arctan} x dx$

Let

$u = \text{arctan} x, \qquad dv = x dx = d\left(\frac{x^2}{2}\right)$

then

$du = d(\text{arctan} x) = (\text{arctan} x)' dx = \frac{1}{1+x^2} dx, \qquad v = \frac{x^2}{2}$

Integrating by parts, we get

$\int x \text{ arctan} x dx = \text{arctan} x \cdot \frac{x^2}{2} - \int \frac{x^2}{2}\cdot\frac{1}{1+x^2} dx = \frac{x^2}{2}\text{arctan} x - \frac{1}{2}\int \frac{x^2}{1+x^2} dx$

$= \frac{x^2}{2}\text{arctan} x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2} dx = \frac{x^2}{2}\text{arctan} x - \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right) dx$

$= \frac{x^2}{2}\text{arctan} x - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{1}{1+x^2} dx = \frac{x^2}{2}\text{arctan} x - \frac{1}{2}x + \frac{1}{2}\text{arctan} x + C$

3) $\int \frac{\ln x}{(1+x)^2} dx$

Let

$u = \ln x, \qquad dv = \frac{1}{(1+x)^2} dx = \left(-\frac{1}{(1+x)}\right)' dx = d\left(-\frac{1}{(1+x)}\right)$

then

$du = d\ln x = (\ln x)' dx = \frac{1}{x} dx, \qquad v = -\frac{1}{(1+x)}$

Integrating by parts, we get

$\int \frac{\ln x}{(1+x)^2} dx = \ln x \cdot \left(-\frac{1}{(1+x)}\right) - \int \left(-\frac{1}{(1+x)}\right) \cdot \frac{1}{x} dx = -\frac{\ln x}{(1+x)} + \int \frac{1}{x(1+x)} dx$

Decompose the fraction

$\frac{1}{x(1+x)} = \frac{A}{x} + \frac{B}{1+x} = \frac{A+Ax+Bx}{x(1+x)}$

The coefficients of like terms in the numerators must be equal:

coefficients of x: $A + B = 0$

constant terms: $A = 1$

Then we get $A = 1$, $B = -1$ and

$\frac{1}{x(1+x)} = \frac{1}{x} - \frac{1}{1+x}$

Integral becomes

$\int \frac{\ln x}{(1+x)^2} dx = -\frac{\ln x}{(1+x)} + \int \left(\frac{1}{x} - \frac{1}{1+x}\right) dx = -\frac{\ln x}{(1+x)} + \int \frac{1}{x} dx - \int \frac{1}{1+x} dx$

$= -\frac{\ln x}{(1+x)} + \ln x - \ln(1+x) + C$

4) $\int (\ln x)^2 dx$

Let

$u = \ln^2 x, \qquad dv = dx$

then

$du = d(\ln^2 x) = (\ln^2 x)' dx = 2\ln x \cdot (\ln x)' \cdot dx = 2\ln x \cdot \frac{1}{x} \cdot dx, \qquad v = x$

Integrating by parts, we get

$\int (\ln x)^2 dx = \ln^2 x \cdot x - \int x \cdot 2\ln x \cdot \frac{1}{x} \cdot dx = x\ln^2 x - 2\int \ln x dx$

Integrate by parts one more time

Let

$u = \ln x, dv = dx$

then

$du = d(\ln x) = (\ln x)' dx = \frac{1}{x} dx, v = x$

We get

$$\int (\ln x)^2 dx = x\ln^2 x - 2\int \ln x dx = x\ln^2 x - 2\left(\ln x \cdot x - \int x \cdot \frac{1}{x} dx\right) = x\ln^2 x - 2x\ln x + 2\int dx$$
$$= x\ln^2 x - 2x\ln x + 2x + C$$

Answer:

1) $\int x\sin 2x dx = -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x + C$

2) $\int x \arctan x dx = \frac{x^2}{2}\arctan x - \frac{1}{2}x + \frac{1}{2}\arctan x + C$

3) $\int \frac{\ln x}{(1+x)^2} dx = -\frac{\ln x}{(1+x)} + \ln x - \ln(1+x) + C$

4) $\int (\ln x)^2 dx = x\ln^2 x - 2x\ln x + 2x + C$

Answer provided by https://www.AssignmentExpert.com