Answer on Question #70850 – Math – Calculus

Question

1. Integrate by substitution

$$\int \sqrt{1+x^2}x^5\,dx$$

Solution

$$\int \sqrt{1+x^2}x^5\,dx =$$

| substitution $u = 1 + x^2,$ |
| $du = 2xdx; x^4 = (u-1)^2$ |

$$= \frac{1}{2}\int u^{\frac{1}{2}}(u-1)^2\,du = \frac{1}{2}\int u^{\frac{1}{2}}(u^2-2u+1)\,du =$$

$$= \frac{1}{2}\int u^{\frac{5}{2}}\,du - \int u^{\frac{3}{2}}\,du + \frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{2}\left(\frac{2}{7}\right)u^{\frac{7}{2}} - \frac{2}{5}u^{\frac{5}{2}} + \frac{1}{2}\left(\frac{2}{3}\right)u^{\frac{3}{2}} + C =$$

$$= \frac{1}{7}(1+x^2)^3\sqrt{1+x^2} - \frac{2}{5}(1+x^2)^2\sqrt{1+x^2} + \frac{1}{3}(1+x^2)\sqrt{1+x^2} + C,$$

where C is an integration constant.

Answer: $\frac{1}{7}(1+x^2)^3\sqrt{1+x^2} - \frac{2}{5}(1+x^2)^2\sqrt{1+x^2} + \frac{1}{3}(1+x^2)\sqrt{1+x^2} + C$

Question

2. Integrate by substitution

$$\int \sec x\,dx, \quad u = \sec x + \tan x$$

Solution

$$\int \sec x\,dx =$$

|substitution $u = \sec x + \tan x = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \frac{1+\sin x}{\cos x},$ |
|$du = \left(-\frac{-\sin x}{\cos^2 x} + \frac{1}{\cos^2 x}\right)dx = \frac{1+\sin x}{\cos x}\cdot \sec x\,dx$ |
| $\sec x\,dx = \frac{du}{u}$ |

$$= \int \frac{du}{u} = \ln|u| + C = \ln|\sec x + \tan x| + C,$$

where C is an integration constant.

Answer: $\ln|\sec x + \tan x| + C.$